មេរៀនទី១: លីមីតនៃស៊ីត

ដំណោះស្រាយលំហាត់

1. តើស៊ីត (u_n) ដែលមានតួទូទៅដូចខាងក្រោមជាស៊ីតរួម ឬរីក?

ក. $u_n = 3n^2 + 5n + 1$

ខ. $u_n = \dfrac{n^2 + n}{2n^2 + 5}$

គ. $u_n = \dfrac{\sin 2n}{5^n}$

ឃ. $u_n = \dfrac{2n}{n+3} + \dfrac{3n^3}{n^2+5}$

ង. $u_n = \dfrac{n \sin n}{n^2 + 1}$

ច. $u_n = 2 - \dfrac{3}{n} + \dfrac{4}{\sqrt{n}}$

ក. យើងមាន $u_n = 3n^2 + 5n + 1$ នោះយើងបាន

$$\lim_{n\to+\infty} u_n = \lim_{n\to+\infty}\left(3n^2 + 5n + 1\right) = +\infty$$

ដូច្នេះ $\boxed{(u_n) \text{ ជាស៊ីតរីក}}$

ខ. យើងមាន $u_n = \dfrac{n^2 + n}{2n^2 + 5}$ នោះយើងបាន

$$\lim_{n\to+\infty} u_n = \lim_{n\to+\infty} \frac{n^2 + n}{2n^2 + 5} = \lim_{n\to+\infty} \frac{1 + \frac{1}{n}}{2 + \frac{5}{n^2}} = \frac{1}{2}$$

ដូច្នេះ $\boxed{(u_n) \text{ ជាស៊ីតរួម}}$

គ. យើងមាន $u_n = \dfrac{\sin 2n}{5^n}$

ដោយ $\forall n \in \mathbb{N},\quad -1 \le \sin 2n \le 1$ នាំឱ្យ $-\dfrac{1}{5^n} \le \dfrac{\sin 2n}{5^n} \le \dfrac{1}{5^n}$

តែ $\lim\limits_{n\to+\infty}\left(-\dfrac{1}{5^n}\right) = \lim\limits_{n\to+\infty}\left(\dfrac{1}{5^n}\right) = 0$ នាំឱ្យ $\lim\limits_{n\to+\infty} \dfrac{\sin 2n}{5^n} = 0$

ដូច្នេះ $\boxed{(u_n) \text{ ជាស៊ីតរួម}}$

ឃ. យើងមាន $u_n = \frac{2n}{n+3} + \frac{3n^3}{n^2+5}$ នោះយើងបាន

$$\lim_{n\to+\infty} u_n = \lim_{n\to+\infty}\left(\frac{2n}{n+3} + \frac{3n^3}{n^2+5}\right) = \lim_{n\to+\infty}\frac{2n}{n+3} + \lim_{n\to+\infty}\frac{3n^3}{n^2+5}$$

$$= \lim_{n\to+\infty}\frac{2}{1+\frac{3}{n}} + \lim_{n\to+\infty}\frac{3n}{1+\frac{5}{n}} = 2 + \lim_{n\to+\infty} 3n = +\infty$$

ដូច្នេះ $\boxed{(u_n)\text{ ជាស្វ៊ីតរីក}}$

ង. យើងមាន $u_n = \frac{n\sin n}{n^2+1} = \frac{\sin n}{n+\frac{1}{n}}$

ដោយ $\forall n\in\mathbb{N},\quad -1\le \sin n \le 1$ នាំឱ្យ $-\frac{1}{n+\frac{1}{n}} \le \frac{\sin n}{n+\frac{1}{n}} \le \frac{1}{n+\frac{1}{n}}$

តែ $\lim_{n\to+\infty}\left(-\frac{1}{n+\frac{1}{n}}\right) = \lim_{n\to+\infty}\left(\frac{1}{n+\frac{1}{n}}\right) = 0$ នោះយើងបាន

$$\lim_{n\to+\infty}\frac{n\sin n}{n^2+1} = 0$$

ដូច្នេះ $\boxed{(u_n)\text{ ជាស្វ៊ីតរួម}}$

ច. យើងមាន $u_n = 2 - \frac{3}{n} + \frac{4}{\sqrt{n}}$ នោះយើងបាន

$$\lim_{n\to+\infty} u_n = \lim_{n\to+\infty}\left(2 - \frac{3}{n} + \frac{4}{\sqrt{n}}\right) = 2 - 0 + 0 = 2$$

ដូច្នេះ $\boxed{(u_n)\text{ ជាស្វ៊ីតរួម}}$

2. គណនាលីមីតនៃស្វ៊ីតខាងក្រោម:

ក. $\lim_{n\to+\infty}\frac{n^2+3n-1}{8n^2-n+1}$

ខ. $\lim_{n\to+\infty}\frac{5n^3+n^2-n}{n^2+n-1}$

គ. $\lim_{n\to+\infty}\frac{5n^3+(-1)^n}{n+(-1)^n}$

ឃ. $\lim_{n\to+\infty}\frac{n^2+\sin n}{5n^2+\cos\pi n}$

ង. $\lim_{n\to+\infty}\left(n^2-\cos^2\pi n\right)$

ច. $\lim_{n\to+\infty}\left[-5n^3+(-1)^n n^3\right]$

ក. $\lim\limits_{n\to+\infty}\dfrac{n^2+3n-1}{8n^2-n+1}=\lim\limits_{n\to+\infty}\dfrac{1+\dfrac{3}{n}-\dfrac{1}{n^2}}{8-\dfrac{1}{n}+\dfrac{1}{n^2}}=\boxed{\dfrac{1}{8}}$

ខ. $\lim\limits_{n\to+\infty}\dfrac{5n^3+n^2-n}{n^2+n-1}=\lim\limits_{n\to+\infty}\dfrac{5n+1-\dfrac{1}{n}}{1+\dfrac{1}{n}-\dfrac{1}{n^2}}=\boxed{+\infty}$

គ. $\lim\limits_{n\to+\infty}\dfrac{5n^3+(-1)^n}{n+(-1)^n}=\lim\limits_{n\to+\infty}\dfrac{5n^2+\dfrac{(-1)^n}{n}}{1+\dfrac{(-1)^n}{n}}=\boxed{+\infty}$

ឃ. $\lim\limits_{n\to+\infty}\dfrac{n^2+\sin n}{5n^2+\cos\pi n}=\lim\limits_{n\to+\infty}\dfrac{1+\dfrac{\sin n}{n^2}}{5+\dfrac{\cos\pi n}{n^2}}$

ដោយ $\lim\limits_{n\to+\infty}\dfrac{\sin n}{n^2}=0$ និង $\lim\limits_{n\to+\infty}\dfrac{\cos\pi n}{n^2}=0$ នោះយើងបាន

ដូច្នេះ $\boxed{\lim\limits_{n\to+\infty}\dfrac{n^2+\sin n}{5n^2+\cos\pi n}=\dfrac{1+0}{5+0}=\dfrac{1}{5}}$

ង. $\lim\limits_{n\to+\infty}\left(n^2-\cos^2\pi n\right)=\lim\limits_{n\to+\infty}\left[n^2\left(1-\dfrac{\cos^2\pi n}{n^2}\right)\right]$

ដោយ $\lim\limits_{n\to+\infty}n^2=+\infty$ និង $\lim\limits_{n\to+\infty}\dfrac{\cos^2\pi n}{n^2}=0$

$\lim\limits_{n\to+\infty}\left[n^2\left(1-\dfrac{\cos^2\pi n}{n^2}\right)\right]=+\infty$

ដូច្នេះ $\boxed{\lim\limits_{n\to+\infty}\left(n^2-\cos^2\pi n\right)=+\infty}$

ច. $\lim\limits_{n\to+\infty}\left[-5n^3+(-1)^n n^3\right]=\lim\limits_{n\to+\infty}n^3\left[-5+(-1)^n\right]=\boxed{-\infty}$

3. គណនាលីមីតនៃស្វ៊ីតខាងក្រោម:

ក. $\lim\limits_{n\to+\infty}\left(\sqrt{n+1}-\sqrt{n}\right)$ ខ. $\lim\limits_{n\to+\infty}\sqrt{n}\left(\sqrt{n-3}-\sqrt{n}\right)$

គ. $\lim\limits_{n\to+\infty}\left(n\sqrt{n^2+1}-1\right)$ ឃ. $\lim\limits_{n\to+\infty}\left(\frac{n!}{(n+1)!}-\frac{2}{n}+3\right)$

ក. $\lim\limits_{n\to+\infty}\left(\sqrt{n+1}-\sqrt{n}\right)$

ដោយ $\sqrt{n+1}-\sqrt{n}=\frac{\left(\sqrt{n+1}-\sqrt{n}\right)\left(\sqrt{n+1}+\sqrt{n}\right)}{\sqrt{n+1}+\sqrt{n}}=\frac{n+1-n}{\sqrt{n+1}+\sqrt{n}}=\frac{1}{\sqrt{n+1}+\sqrt{n}}$

នោះយើងបាន $\lim\limits_{n\to+\infty}\left(\sqrt{n+1}-\sqrt{n}\right)=\lim\limits_{n\to+\infty}\frac{1}{\sqrt{n+1}+\sqrt{n}}=0$

ដូច្នេះ $\boxed{\lim\limits_{n\to+\infty}\left(\sqrt{n+1}-\sqrt{n}\right)=0}$

ខ. $\lim\limits_{n\to+\infty}\sqrt{n}\left(\sqrt{n-3}-\sqrt{n}\right)$

ដោយ $\sqrt{n}\left(\sqrt{n-3}-\sqrt{n}\right)=\frac{\sqrt{n}\left(\sqrt{n-3}-\sqrt{n}\right)\left(\sqrt{n-3}+\sqrt{n}\right)}{\sqrt{n-3}+\sqrt{n}}=\frac{-3}{\sqrt{1-\frac{3}{n}}+1}$

ដូច្នេះ $\lim\limits_{n\to+\infty}\sqrt{n}\left(\sqrt{n-3}-\sqrt{n}\right)=\lim\limits_{n\to+\infty}\left(-\frac{3}{\sqrt{1-\frac{3}{n}}+1}\right)=\boxed{-\frac{3}{2}}$

គ. $\lim\limits_{n\to+\infty}n\left(\sqrt{n^2+1}-1\right)=\lim\limits_{n\to+\infty}\left[n^2\left(\sqrt{1+\frac{1}{n^2}}-1\right)\right]$

$=\lim\limits_{n\to+\infty}\left[n^2\left(\sqrt{1+\frac{1}{n^2}}-\frac{1}{n^2}\right)\right]=\boxed{+\infty}$

ឃ. $\lim\limits_{n\to+\infty}\left(\frac{n!}{(n+1)!-n!}-\frac{2}{3}+3\right)$

ដោយ $\frac{n!}{(n+1)!-n!}-\frac{2}{n}+3=\frac{n!}{(n+1)n!-n!}-\frac{2}{n}+3=\frac{1}{n}-\frac{2}{n}+3=\frac{-1}{n}+3$

ដូច្នេះ $\lim\limits_{n\to+\infty}\left(\frac{n!}{(n+1)!-n!}-\frac{2}{n}+3\right)=\lim\limits_{n\to+\infty}\left(3-\frac{1}{n}\right)=\boxed{3}$

4. គេមានស្វ៊ីតដែលកំណត់ចំពោះគ្រប់ $n \in \mathrm{N}$ មានតួទូទៅដូចខាងក្រោម:

$u_n = \dfrac{n^3}{2^n}$, $v_n = \dfrac{2^n}{n!}$ ដែល $n! = n(n-1)......2.1$

ក. គណនា $\lim\limits_{n\to+\infty} \dfrac{u_{n+1}}{u_n}$ និង $\lim\limits_{n\to+\infty} \dfrac{v_{n+1}}{v_n}$ ខ. គណនា $\lim\limits_{n\to+\infty} \dfrac{2^n + n^3}{n! + n^3}$

ក. គណនា

$$\bullet \lim_{n\to+\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to+\infty} \frac{\dfrac{(n+1)^3}{2^{n+1}}}{\dfrac{n^3}{2^n}} = \lim_{n\to+\infty} \frac{(n+1)^3}{2^{n+1}} \cdot \frac{2^n}{n^3} = \lim_{n\to+\infty} \left[\left(\frac{n+1}{n} \right)^3 \frac{1}{2} \right] = \lim_{n\to+\infty} \left[\frac{1}{2} \left(1 + \frac{1}{n} \right)^3 \right] = \boxed{\frac{1}{2}}$$

$$\bullet \lim_{n\to+\infty} \frac{v_{n+1}}{v_n} = \lim_{n\to+\infty} \frac{\dfrac{2^{n+1}}{(n+1)!}}{\dfrac{2^n}{n!}} = \lim_{n\to+\infty} \left[\frac{2^{n+1}}{(n+1)!} \frac{n!}{2^n} \right] = \lim_{n\to+\infty} \left[\frac{n!}{(n+1)n!} \frac{2^n 2}{2^n} \right] = \lim_{n\to+\infty} \frac{2}{n+1} = \boxed{0}$$

ខ. គណនា $\lim\limits_{n\to+\infty} \dfrac{2^n + n^3}{n! + n^3} = \lim\limits_{n\to+\infty} \dfrac{2^n}{n!} \left(\dfrac{1 + \dfrac{n^3}{2^n}}{1 + \dfrac{n^3}{n!}} \right) = 0$ ព្រោះ $\lim\limits_{n\to+\infty} \dfrac{2^n}{n!} = 0$

ដូច្នេះ $\boxed{\lim\limits_{n\to+\infty} \dfrac{2^n + n^3}{n! + n^3} = 0}$

5. គណនាលីមីតនៃស្វ៊ីត (a_n) ដែលស្គាល់ដូចតទៅ

ក. $a_1 = 2$, $a_{n+1} = \dfrac{1}{2} a_n + 3$ ខ. $a_1 = 3$, $a_{n+1} = 2a_n - 5$

គ. $a_1 = 1$, $a_{n+1} = \dfrac{1}{3} a_n + \dfrac{4}{3}$

ក. យើងមាន $a_1 = 2$, $a_{n+1} = \dfrac{1}{2} a_n + 3$ នោះយើងបានសមីការសម្គាល់នៃស្វ៊ីតគឺ

$c = \dfrac{1}{2} c + 3 \Rightarrow c = 6$ យើងបាន

$a_{n+1} - 6 = \dfrac{1}{2}(a_n - 6) \Leftrightarrow \dfrac{a_{n+1} - 6}{a_n - 6} = \dfrac{1}{2}$

តាង $b_n = a_n - 6$ ដែលជាស្វ៊ីតធរណីមាត្រមានផលធៀបស្មើនឹង $\dfrac{1}{2}$

យើងបាន

$$b_1 = a_1 - 6 = 2 - 6 = -4$$

$$b_n = b_1 q^{n-1} = (-4)\left(\frac{1}{2}\right)^{n-1}$$

$$\Rightarrow a_n = b_n + 6 = (-4)\left(\frac{1}{2}\right)^{n-1}$$

ដូច្នេះ $\boxed{\lim_{n\to+\infty} a_n = \lim_{n\to+\infty}\left[(-4)\left(\frac{1}{2}\right)^{n-1} + 6\right] = 6}$

ខ. យើងមាន $a_1 = 3$, $a_{n+1} = 2a_n - 5$ យើងបានសមីការសម្គាល់នៃស្វ៊ីតគឺ

$$c = 2c - 5 \Rightarrow c = 5$$

យើងបាន

$$a_{n+1} - 5 = 2(a_n - 5) \Leftrightarrow \frac{a_{n+1} - 5}{a_n - 5} = 2$$

តាង $b_n = a_n - 5$ ជាស្វ៊ីតធរណីមាត្រមានផលធៀបរួមស្មើនឹង 2

យើងបាន

$$b_1 = a_1 - 5 = 3 - 5 = -2$$

$$b_n = b_1 q^{n-1} = (-2)2^{n-1} = -2^n$$

$$\Rightarrow a_n = b_n + 5 = -2^n + 5$$

ដូច្នេះ $\boxed{\lim_{n\to+\infty} a_n = \lim_{n\to+\infty}\left(5 - 2^n\right) = -\infty}$

គ. យើងមាន $a_1 = 1$, $a_{n+1} = \frac{1}{3}a_n + \frac{4}{3}$ យើងបានសមីការសម្គាល់នៃស្វ៊ីតគឺ

$$c = \frac{1}{3}c + \frac{4}{3} \Rightarrow c = 2$$

យើងបាន

$$a_{n+1} - 2 = \frac{1}{3}(a_n - 2) \Leftrightarrow \frac{a_{n+1} - 2}{a_n - 2} = \frac{1}{3}$$

តាង $b_n = a_n - 2$ ជាស្វ៊ីតធរណីមាត្រមានផលធៀបរួមស្មើនឹង $\frac{1}{3}$

យើងបាន

$$b_1 = a_1 - 2 = 1 - 2 = -1$$

$$b_n = b_1 q^{n-1} = (-1)\left(\frac{1}{3}\right)^{n-1}$$

$$\Rightarrow a_n = b_n + 2 = (-1)\left(\frac{1}{3}\right)^{n-1} + 2$$

ដូច្នេះ $\boxed{\lim_{n\to+\infty} a_n = \lim_{n\to+\infty}\left[(-1)\left(\frac{1}{3}\right)^{n-1} + 2\right] = 2}$

6. ពិនិត្យស៊េរីខាងក្រោមនេះ តើជាស៊េរីរួម ឬរីក?

ក. $\sum_{n=0}^{\infty} 3\left(\frac{3}{2}\right)^n$ ខ. $\sum_{n=0}^{\infty} 1000(1.055)^n$

គ. $\sum_{n=1}^{\infty} \frac{n+\sqrt{n}}{2n^3-1}$ ឃ. $\sum_{n=1}^{\infty} \frac{2^n+1}{2^{n+1}}$

ង. $2 + \frac{3}{2} + \frac{9}{8} + \frac{27}{32} + \frac{81}{128} + \cdots\cdots$ ច. $\sum_{n=1}^{\infty}\left(\sqrt{2n+1} - \sqrt{2n-1}\right)$

ក. យើងមាន $\sum_{n=0}^{\infty} 3\left(\frac{3}{2}\right)^n = 3 + 3\frac{3}{2} + 3\left(\frac{3}{2}\right)^2 + \cdots\cdots$ ជាស៊េរីធរណីមាត្រមានផលធៀបរួមស្មើនឹង $\frac{3}{2} > 1$

យើងបាន $\sum_{n=0}^{\infty} 3\left(\frac{3}{2}\right)^n$ ជាស៊េរីរីក

ដូច្នេះ $\boxed{\sum_{n=0}^{\infty} 3\left(\frac{3}{2}\right)^n \text{ ជាស៊េរីរីក}}$

ខ. យើងមាន $\sum_{n=0}^{\infty} 1000(1.055)^n = 1000 + 1000\times1.055 + 1000.(1.055)^2 + \cdots$ ជាស៊េរីធរណីមាត្រមានផលធៀបរួមស្មើ $1.055 > 1$

យើងបាន $\sum_{n=0}^{\infty} 1000(1.055)$ ជាស៊េរីរីក

ដូច្នេះ $\boxed{\sum_{n=0}^{\infty} 1000(1.055)^n \text{ ជាស៊េរីរីក}}$

គ. យើងមាន $\sum_{n=1}^{\infty} \frac{n+\sqrt{n}}{2n^3-1}$

យើងបាន

$$\forall n \ge 1,\ a_n = \frac{n+\sqrt{n}}{2n^3-1} < \frac{2n}{2n^3-2n} = \frac{1}{n(n-1)} = \frac{1}{n-1} - \frac{1}{n}$$

ពិនិត្យស៊េរី $\sum_{n=2}^{\infty}\left(\frac{1}{n-1}-\frac{1}{n}\right)=\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots=1$ ជាស៊េរីរួម

យើងបាន $\sum_{n=1}^{\infty}a_n=\sum_{n=1}^{\infty}\frac{n+\sqrt{n}}{2n^3-1}=2+\sum_{n=2}^{\infty}\frac{n+\sqrt{n}}{2n^3-1}<2+\sum_{n=2}^{\infty}\left(\frac{1}{n-1}-\frac{1}{n}\right)=3$

$\Rightarrow \sum_{n=1}^{\infty}\frac{n+\sqrt{n}}{2n^3-1}$ ជាស៊េរីរួម

ដូច្នេះ $\boxed{\text{ស៊េរី } \sum_{n=1}^{\infty}\frac{n+\sqrt{n}}{2n^3-1} \text{ ជាស៊េរីរួម។}}$

ឃ. យើងមាន $\sum_{n=1}^{\infty}\frac{2^n+1}{2^{n+1}}=\sum_{n=1}^{\infty}\frac{2^n}{2^{n+1}}+\sum_{n=1}^{\infty}\frac{1}{2^{n+1}}=\frac{1}{2}\sum_{n=1}^{\infty}1^n+\frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n$

$$S_n=\frac{1}{2}\left(\sum_{k=1}^{n}1^k+\sum_{k=1}^{n}\left(\frac{1}{2}\right)^k\right)=\frac{1}{2}\left[n+\left(\frac{1}{2}+\frac{1}{2}\frac{1}{2}+\frac{1}{2}\left(\frac{1}{2}\right)^2+\cdots+\frac{1}{2}\left(\frac{1}{2}\right)^n\right)\right]$$

យើងបាន $=\frac{1}{2}\left[n+\left(\frac{\frac{1}{2}}{1-\frac{1}{2}}\right)\right]=\frac{1}{2}[n+1]$

$$\Rightarrow \lim_{n\to+\infty}S_n=\lim_{n\to+\infty}\frac{n+1}{2}=+\infty$$

ដូច្នេះ $\boxed{\sum_{n=1}^{\infty}\frac{2^n+1}{2^{n+1}} \text{ ជាស៊េរីរីក}}$

ង. យើងមាន $2+\frac{3}{2}+\frac{9}{8}+\frac{27}{32}+\frac{81}{128}+\cdots=\sum_{n=1}^{\infty}2\left(\frac{3}{4}\right)^n$

ជាស៊េរីធរណីមាត្រមានផលធៀបរួមស្មើ $\frac{3}{4}<1$

យើងបានស៊េរីនេះជាស៊េរីរួមរក $\frac{2}{1-\frac{3}{4}}=8$

ដូច្នេះ $\boxed{\sum_{n=0}^{\infty}2\left(\frac{3}{4}\right)^n \text{ ជាស៊េរីរួមរក } 8\text{។}}$

ច. យើងមាន

$$\sum_{n=1}^{\infty}\left(\sqrt{2n+1}-\sqrt{2n-1}\right)=\left(\sqrt{3}-\sqrt{1}\right)+\left(\sqrt{5}-\sqrt{3}\right)+\left(\sqrt{7}-\sqrt{5}\right)+\cdots$$

$$\Rightarrow S_n=\left(\sqrt{3}-\sqrt{1}\right)+\left(\sqrt{5}-\sqrt{3}\right)+\left(\sqrt{7}-\sqrt{5}\right)+\cdots+\left(\sqrt{2n+1}-\sqrt{2n-1}\right)$$

$$=\sqrt{2n+1}-1$$

$$\Rightarrow \lim_{n\to+\infty} S_n=\lim_{n\to+\infty}\left(\sqrt{2n+1}-1\right)=+\infty$$

ដូច្នេះ $\boxed{\sum_{n=1}^{\infty}\left(\sqrt{2n+1}-\sqrt{2n-1}\right) \text{ ជាស៊េរីរីក}}$

7. រកផលបូកនៃស៊េរីរួមខាងក្រោម:

ក. $1+0.1+0.01+0.001+\cdots$

ខ. $\sum_{n=1}^{\infty}\frac{2}{4n^2-1}$

គ. $\sum_{n=1}^{\infty}\frac{1}{(2n-1)(2n+1)}$

ឃ. $\sum_{n=0}^{\infty}2\left(\cos\frac{\pi}{3}\right)^n$

ង. $\sum_{n=1}^{\infty}\frac{5(-1)^n}{4^n}$

ច. $\frac{2}{3}+\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^3+\cdots$

ក. យើងមាន

$1+0.1+0.01+0.001+\cdots=\sum_{n=0}^{\infty}\left(\frac{1}{10}\right)^n$ ជាស៊េរីធរណីមាត្រមានផលធៀបរួមស្មើ $\frac{1}{10}$ និងតួទីមួយស្មើ 1

យើងបាន

$$S_n=\sum_{k=0}^{n}\left(\frac{1}{10}\right)^k=\frac{1-\left(\frac{1}{10}\right)^n}{1-\left(\frac{1}{10}\right)}=\frac{10}{9}\left[1-\left(\frac{1}{10}\right)^n\right]$$

$$\Rightarrow \lim_{n\to+\infty} S_n=\lim_{n\to+\infty}\frac{10}{9}\left[1-\left(\frac{1}{10}\right)^n\right]=\frac{10}{9}$$

ដូច្នេះ $\boxed{\sum_{n=0}^{\infty}\left(\frac{1}{10}\right)^n=\frac{10}{9}}$

ខ. យើងមាន

$$\sum_{n=1}^{\infty}\frac{2}{4n^2-1}=2\sum_{n=1}^{\infty}\frac{1}{2}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)=\sum_{n=1}^{\infty}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$$

$$\Rightarrow S_n=\sum_{k=1}^{n}\left(\frac{1}{2k-1}-\frac{1}{2k+1}\right)=\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\cdots+\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$$

$$=1-\frac{1}{2n+1}$$

$$\Rightarrow \lim_{n\to+\infty}S_n=\lim_{n\to+\infty}\left(1-\frac{1}{2n+1}\right)=1$$

ដូច្នេះ $\boxed{\sum_{n=1}^{\infty}\frac{2}{4n^2-1}=1}$

គ. យើងមាន

$$\sum_{n=1}^{\infty}\frac{1}{(2n-1)(2n+1)}=\sum_{n=}^{\infty}\frac{1}{2}\left[\frac{1}{2n-1}-\frac{1}{2n+1}\right]=\frac{1}{2}\left[\left(1-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\cdots\right]$$

$$\Rightarrow S_n=\frac{1}{2}\left[\left(1-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\cdots+\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)\right]=\frac{1}{2}\left(1-\frac{1}{2n+1}\right)$$

$$\Rightarrow \lim_{n\to+\infty}\frac{1}{2}\left(1-\frac{1}{2n+1}\right)=\frac{1}{2}$$

ដូច្នេះ $\boxed{\sum_{n=1}^{\infty}\frac{1}{(2n-1)(2n+1)}=\frac{1}{2}}$

ឃ. យើងមាន $\sum_{n=0}^{\infty}2\left(\cos\frac{\pi}{3}\right)^n$

ដោយ $\cos\frac{\pi}{3}=\frac{1}{2}$ នោះ

យើងបាន

$$\sum_{n=0}^{\infty}2\left(\cos\frac{\pi}{3}\right)^n=\sum_{n=0}^{\infty}2\left(\frac{1}{2}\right)^n$$ ជាស៊េរីធរណីមាត្រមានផលធៀបរួមស្មើ $\frac{1}{2}$ និងតួទីមួយស្មើ 2

យើងបាន

$$S=\sum_{n=0}^{\infty}2\left(\frac{1}{2}\right)^n=\frac{2}{1-\frac{1}{2}}=4$$

ដូច្នេះ $\boxed{\sum_{n=0}^{\infty}2\left(\cos\frac{\pi}{3}\right)^n=4}$

ង. យើងមាន

$\sum_{n=1}^{\infty}\frac{5(-1)^n}{4^n}=-\frac{5}{4}+\frac{5}{4^2}-\frac{5}{4^3}+\cdots$ ជាសេរីធរណីមាត្រមានផលធៀបរួមស្មើនឹង $-\frac{1}{4}$ និងតួទីមួយស្មើនឹង $-\frac{5}{4}$

យើងបាន

$$S=\sum_{n=1}^{\infty}\frac{5(-1)^n}{4^n}=\frac{-\frac{5}{4}}{1+\frac{1}{4}}=-1$$

ដូច្នេះ $\boxed{\sum_{n=1}^{\infty}\left(\frac{5(-1)^n}{4^n}\right)=-1}$

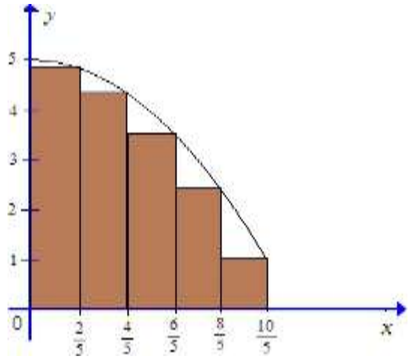
ច. យើងមាន $\frac{2}{3}+\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^3+\cdots=\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$ ជាសេរីធរណីមាត្រ មានផលធៀបរួម ស្មើនឹង $\frac{2}{3}$ និងតួទីមួយស្មើនឹង $\frac{2}{3}$

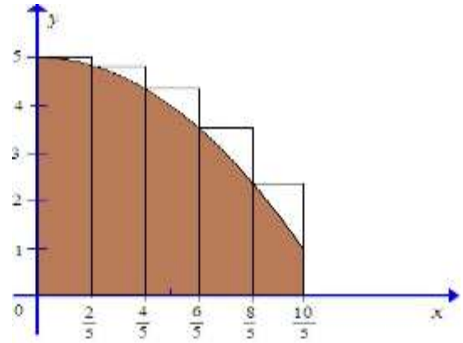
យើងបាន

$$S=\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n=\frac{\frac{2}{3}}{1-\frac{2}{3}}=2$$

ដូច្នេះ $\boxed{\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n=2}$

8. គណនាផ្ទៃក្រឡានៃតំបន់ដែលខណ្ឌដោយចតុកោណទាំងប្រាំ (ដូចរូបទី 1 និង រូបទី 2) ខាងក្រោមរួចប៉ាន់ស្មានផ្ទៃក្រឡានៃផ្នែកប្លង់ដែលខណ្ឌដោយក្រាបតាង $f(x)=-x^2+5$ អ័ក្សអាប់ស៊ីស ហើយត្រូវគ្នានឹងចន្លោះ $[0,2]$ ។





គណនាផ្ទៃក្រឡានៃតំបន់ដែលខណ្ឌដោយចតុកោណកែងទាំងប្រាំ

-ចំពោះរូបទី2

យើងមាន $f(x)=-x^2+5$ ដែល $x\in[0,2]$

យើងបាន $\Delta_x=\frac{2-0}{5}=\frac{2}{5}$ ហើយ $f(x)$ ជាអនុគមន៍មិនអវិជ្ជមានលើ$[0,2]$ យកចំណុច ចុងខាងស្តាំនៃចន្លោះជាចំណុចកំពូលនៃកម្ពស់ចតុកោណកែង

យើងបាន

$$x_i=0+(i-1)\left(\frac{2}{5}\right)=\frac{2}{5}i-\frac{2}{5}$$

នាំឱ្យផ្ទៃក្រឡានៃចតុកោណកែងទាំងប្រាំគឺ

$$S_5=\sum_{i=1}^{5}f\left(\left(\frac{2}{5}i-\frac{2}{5}\right)\right)\frac{2}{5}=\frac{2}{5}\sum_{i=1}^{5}\left[-\left(\frac{2}{5}i-\frac{2}{5}\right)^2+5\right]$$

$$=\frac{2}{5}\sum_{i=1}^{5}\left[-\left(\frac{2}{5}i-\frac{2}{5}\right)^2\right]+\frac{2}{5}\sum_{i=1}^{5}5$$

$$=\frac{2}{5}\left[-\left(\frac{2}{5}-\frac{2}{5}\right)^2-\left(\frac{2\times2}{5}-\frac{2}{5}\right)^2-\left(\frac{2\times3}{5}-\frac{2}{5}\right)^2-\left(\frac{2\times4}{5}-\frac{2}{5}\right)^2-\left(\frac{2\times5}{5}-\frac{2}{5}\right)^2\right]+\frac{2\times25}{5}$$

$$=\frac{2}{5}\left[0-\left(\frac{4}{25}\right)-\left(\frac{16}{25}\right)-\left(\frac{36}{25}\right)-\left(\frac{64}{25}\right)\right]+10$$

$$=8.08$$

-ចំពោះរូបទី1

យើងបាន $\Delta_x=\frac{2-0}{5}=\frac{2}{5}$ ហើយ $f(x)$ជាអនុគមន៍មិនអវិជ្ជមានលើ $[0,2]$យកចំណុចចុងខាងឆ្វេងនៃចន្លោះជាចំណុចកំពូលនៃកម្ពស់ចតុកោណកែង

យើងបាន

$$x_i=0+\frac{2}{5}i=\frac{2}{5}i$$

នាំឱ្យផលបូកផ្ទៃក្រឡានៃចតុកោណកែងទាំងប្រាំគឺ

$$S'_5 = \sum_{i=1}^{5}\left(f\left(\frac{2}{5}i\right)\frac{2}{5}\right) = \frac{2}{5}\sum_{i=1}^{5}\left[-\left(\frac{2}{5}i\right)^2 + 5\right]$$

$$= \frac{2}{5}\sum_{i=1}^{5} -\left(\frac{2}{5}i\right)^2 + \frac{2}{5}\sum_{i=1}^{5} 5$$

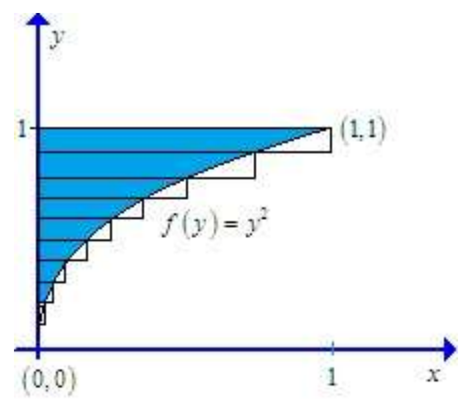
$$= \frac{2}{5}\left[-\left(\frac{2.1}{5}\right)^2 - \left(\frac{2.2}{5}\right)^2 - \left(\frac{2.3}{5}\right)^2 - \left(\frac{2.4}{5}\right)^2 - \left(\frac{2.5}{5}\right)^2\right] + \frac{2}{5}.25$$

$$= \frac{2}{5}\left[-\frac{4}{25} - \frac{16}{25} - \frac{36}{25} - \frac{64}{25} - \frac{100}{25}\right] + 10$$

$$= 6.48$$

យើងឃើញថា $S_5 < S$ ហើយ $S < S'_5$ នោះយើងបាន S ជាផ្ទៃក្រឡាខណ្ឌដោយក្រាប $f(x)$ និងអ័ក្សអាប់ស៊ីសគឺស្ថិតនៅចន្លោះ 6.48 ទៅ 8.08 ។

9. គណនាផ្ទៃក្រឡានៃតំបន់ដែលខណ្ឌដោយក្រាប នៃ $f(x) = y^2$ និងអ័ក្សអរដោនេលើចន្លោះ $[0,1]$ ដូចរូបខាងស្តាំ។



គណនាផ្ទៃក្រឡានៃតបន់ខណ្ឌដោយក្រាប $f(y) = y^2$ និង អ័ក្សអរដោនេលើចន្លោះ $[0,1]$

យើងមាន $f(y) = y^2$

យើងចែកចន្លោះ $[0,1]$ ជា n ចន្លោះយើងបាន $\Delta_y = \frac{1-0}{n} = \frac{1}{n}$ និង $y_i = 0 + \frac{1}{n}i = \frac{i}{n}$

នាំឱ្យផ្ទៃក្រឡានៃតំបន់ខណ្ឌដោយក្រាប និងអ័ក្សអរដោនគឺ

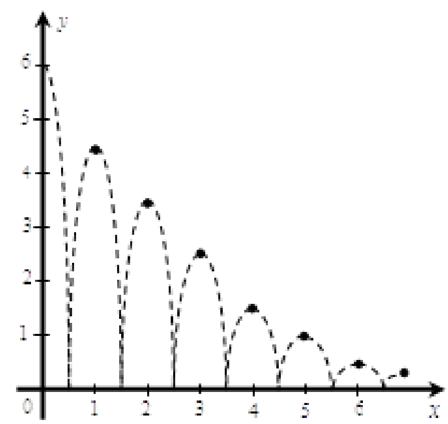
$$S_n=\sum_{i=1}^{n} f\left(\frac{i}{n}\right)\frac{1}{n}=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{i}{n}\right)^2=\frac{1}{n^3}\sum_{i=1}^{n}i^2$$
$$=\frac{1}{n^3}\left(1^2+2^2+3^2+\cdots+n^2\right)$$
$$=\frac{1}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$$

តាង S ជាផលបូកនៃស៊េរី (S_n) នោះយើងបាន

$$S=\lim_{n\to+\infty}S_n=\lim_{n\to+\infty}\frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)=\lim_{n\to+\infty}\frac{\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6}=\frac{1}{3}$$

ដូច្នេះ ផ្ទៃក្រឡានៃតំបន់ខណ្ឌដោយក្រាប និង អ័ក្សអរដោនេគឺ $\frac{1}{3}$ ។

10. បាល់មួយធ្លាក់ចុះពីកម្ពស់ $6m$ ហើយលោត ចុះ ឡើងៗ ដូចរូបខាងស្តាំ។ កម្ពស់ដែលលោតចុះឡើងនីមួយៗស្មើនឹង $\frac{2}{3}$ នៃកម្ពស់ដើម។ រកប្រវែងចម្ងាយសរុបដែលបាល់បានធ្លាក់ចុះឡើងៗ។



គណនាប្រវែងចម្ងាយសរុបដែលបាល់ធ្លាក់ចុះឡើងៗ

យើងមាន $6m$ ជាកម្ពស់ដើម ហើយកម្ពស់ក្រោយនៃការលោតចុះឡើងស្មើ $\frac{2}{3}$ កម្ពស់ដើមនោះយើងតាង

$a_1=6\,,\,a_2=6\frac{2}{3}\,,\,a_3=6\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)=6\left(\frac{2}{3}\right)^2,\cdots,\,a_n=6\left(\frac{2}{3}\right)^{n-1},\cdots$

យើងបានចម្ងាយសរុបគឺ

$$S_n=a_1+a_2+a_3+\cdots+a_n=6+6\left(\frac{2}{3}\right)+6\left(\frac{2}{3}\right)^2+\cdots+6\left(\frac{2}{3}\right)^{n-1}$$

ដោយ (S_n)ជាស៊េរីធរណីមាត្រដែលមានផលធៀបរួមស្មើនឹង $\frac{2}{3}$ និង មានតួទីមួយស្មើនឹង 6 នោះ យើងបាន S ជាផលបូកនៃស៊េរី (S_n)គឺ

$$S=\lim_{n\to+\infty}\sum_{k=0}^{n-1}6\left(\frac{2}{3}\right)^k=\frac{6}{1-\frac{2}{3}}=18$$

ដូច្នេះ ចម្ងាយសរុបនៃបាល់ដែលលោតចុះឡើងៗគឺ $18m$ ។

11. គណនា

ក. $\lim\limits_{n\to+\infty}\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\cdots\left(1-\frac{1}{n^2}\right)$

ខ. $\lim\limits_{n\to+\infty}\left[\frac{n}{\sqrt{n^4+1}}+\frac{n}{\sqrt{n^4+2}}+\cdots\cdots+\frac{n}{\sqrt{n^4+n}}\right]$

ក. $\lim\limits_{n\to+\infty}\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\cdots\left(1-\frac{1}{n^2}\right)$

យើងមាន

$$\begin{aligned}\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\cdots\left(1-\frac{1}{n^2}\right)&=\frac{3}{2^2}\frac{8}{3^2}\cdots\cdots\frac{\left(n^2-1\right)}{n^2}\\&=\frac{3}{2^2}\frac{2\times 4}{3^2}\cdots\cdots\frac{(n-1)(n+1)}{n^2}\\&=\frac{1.3}{2^2}\frac{2.4}{3^2}\frac{3.5}{4^2}\cdots\cdots\frac{(n-1)(n+1)}{n^2}\\&=\frac{1}{2}\frac{(n+1)}{n}=\frac{1}{2}\left(1+\frac{1}{n}\right)\end{aligned}$$

យើងបាន

$$\lim_{n\to+\infty}\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\cdots\left(1-\frac{1}{n^2}\right)=\lim_{n\to+\infty}\frac{1}{2}\left(1+\frac{1}{n}\right)=\frac{1}{2}$$

ដូច្នេះ $\lim\limits_{n\to+\infty}\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\cdots\left(1-\frac{1}{n^2}\right)=\frac{1}{2}$

ខ. គណនា $\lim\limits_{n\to+\infty}\left(\frac{n}{\sqrt{n^4+1}}+\frac{n}{\sqrt{n^4+2}}+\cdots+\frac{n}{\sqrt{n^4+n}}\right)$

យើងតាង $S_n=\frac{n}{\sqrt{n^4+1}}+\frac{n}{\sqrt{n^4+2}}+\cdots+\frac{n}{\sqrt{n^4+n}}$

យើងបាន

$$1\le k\le n,\Rightarrow n^4+1\le n^4+k\le n^4+n$$
$$\Rightarrow \sqrt{n^4+1}\le\sqrt{n^4+k}\le\sqrt{n^4+n}$$
$$\Rightarrow \frac{1}{\sqrt{n^4+n}}\le\frac{1}{\sqrt{n^4+k}}\le\frac{1}{\sqrt{n^4+1}}$$
$$\Rightarrow \frac{n}{\sqrt{n^4+n}}\le\frac{n}{\sqrt{n^4+k}}\le\frac{n}{\sqrt{n^4+1}}$$

$$+\begin{cases}\dfrac{n}{\sqrt{n^4+n}}\le\dfrac{n}{\sqrt{n^4+1}}\le\dfrac{n}{\sqrt{n^4+1}}\\ \dfrac{n}{\sqrt{n^4+n}}\le\dfrac{n}{\sqrt{n^4+2}}\le\dfrac{n}{\sqrt{n^4+1}}\\ \vdots\\ \dfrac{n}{\sqrt{n^4+n}}\le\dfrac{n}{\sqrt{n^4+n}}\le\dfrac{n}{\sqrt{n^4+1}}\end{cases}$$
$$\overline{\frac{n.n}{\sqrt{n^4+n}}\le S_n\le\frac{n.n}{\sqrt{n^4+1}}}$$

តែ $\lim\limits_{n\to+\infty}\dfrac{n^2}{\sqrt{n^4+1}}=\lim\limits_{n\to+\infty}\dfrac{1}{\sqrt{1+\dfrac{1}{n^4}}}=1$ និង $\lim\limits_{n\to+\infty}\dfrac{n^2}{\sqrt{n^4+n}}=\lim\limits_{n\to+\infty}\dfrac{1}{\sqrt{1+\dfrac{1}{n^3}}}=1$

ដូច្នេះ $\lim\limits_{n\to+\infty}S_n=1$

12. គណនា $\lim\limits_{n\to+\infty}\left[(q-1)+q^2\left(q^2-q\right)+q^4\left(q^3-q^2\right)\cdots\cdots+q^{2(n-1)}\left(q^n-q^{n-1}\right)\right]$

ដែល $q=2^{\frac{1}{2}}$ ។

យើងមាន

$$(q-1)+q^2\left(q^2-q\right)+q^4\left(q^3-q^2\right)+\cdots\cdots+q^{2(n-1)}\left(q^n-q^{n-1}\right)$$
$$=(q-1)+q^3(q-1)+q^6(q-1)+\cdots\cdots+q^{3(n-1)}(q-1)$$
$$=(q-1)\left(1+q^3+q^6+\cdots\cdots+q^{3(n-1)}\right)$$
$$=(q-1)\left(\frac{1-\left(q^3\right)^n}{1-q^3}\right)=-(1-q)\left[\frac{1-\left(q^3\right)^n}{(1-q)\left(1+q+q^2\right)}\right]$$
$$=-\frac{1-\left(q^3\right)^n}{1+q+q^2}$$

ដោយ $q=2^{\frac{1}{2}}$ នោះយើងបាន

$$-\frac{1-\left(q^3\right)^n}{1+q+q^2}=-\frac{1-\left(2^{\frac{1}{2}}\right)^{3n}}{1+2^{\frac{1}{2}}+\left(2^{\frac{1}{2}}\right)^2}=\frac{2^{\frac{3n}{2}}-1}{3+\sqrt{2}}=\frac{\left(2^{\frac{3n}{2}}-1\right)\left(3-\sqrt{2}\right)}{7}$$

$$=\frac{\left(2^{\frac{3n}{2}}-1\right)\left(3-\sqrt{2}\right)}{7}$$

យើងបាន $\lim\limits_{n\to+\infty}\left(3-\sqrt{2}\right)\left(2^{\frac{3n}{2}}-1\right)=+\infty$

ដូច្នេះ $\lim\limits_{n\to+\infty}\left[(q-1)+q^2\left(q^2-q\right)+q^4\left(q^3-q^2\right)+\cdots\cdots+q^{2(n-1)}\left(q^n-q^{n-1}\right)\right]=+\infty$

ចំពោះ $q=2^{\frac{1}{2}}$

13. គេឱ្យស្វ៊ីត (a_n) កំណត់ដោយ $a_n=\dfrac{1}{2+q^n}$ ដែល $q\neq-1$ ។ សិក្សាលីមីតនៃស្វ៊ីត (a_n) កាលណា $n\to+\infty$

សិក្សាលីមីតនៃស្វ៊ីត (a_n) កាលណា $n\to+\infty$

-បើ $q>1$ នោះគេបាន $\lim\limits_{n\to+\infty}q^n=+\infty$

ដូច្នេះ $\lim\limits_{n\to+\infty}a_n=\lim\limits_{n\to+\infty}\dfrac{1}{2+q^n}=0$

-បើ $q=1$ នោះគេបាន $\lim\limits_{n\to+\infty}q^n=1$

ដូច្នេះ $\lim\limits_{n\to+\infty}a_n=\lim\limits_{n\to+\infty}\dfrac{1}{2+q^n}=\dfrac{1}{3}$

-បើ $-1<q<1$ នោះគេបាន $\lim\limits_{n\to+\infty}q^n=0$

ដូច្នេះ $\lim\limits_{n\to+\infty}a_n=\lim\limits_{n\to+\infty}\dfrac{1}{2+q^n}=\dfrac{1}{2}$

-បើ $q<-1$ នោះគេបាន $\lim\limits_{n\to+\infty}|q|^n=+\infty$

ដូច្នេះ $\lim\limits_{n\to+\infty}a_n=\lim\limits_{n\to+\infty}\dfrac{1}{2+q^n}=0$

14. ចំពោះគ្រប់ $n \in \mathbb{N}$ គេបាន $S_n = \frac{2}{1.3} + \frac{2}{3.5} + \frac{2}{5.7} + \cdots\cdots + \frac{2}{(2n+1)(2n+3)} = \sum_{p=0}^{n} \frac{2}{(2n+1)(2n+3)}$

ក. គណនា S_n ជាអនុគមន៍នៃ n ដោយប្រើ $\frac{2}{(2p+1)(2p+3)}$ ជាទម្រង់

$\frac{a}{2p+1} + \frac{b}{2p+3}$ ។

ខ. គណនា $\lim_{n\to+\infty} S_n$ ។

ក. គណនា S_n ជាអនុគមន៍នៃ n

ដោយប្រើ $\frac{2}{(2p+1)(2p+3)} = \frac{a}{2p+1} + \frac{b}{2p+3}$

គេបាន

$$\frac{2}{(2p+1)(2p+3)} = \frac{a(2p+3)}{(2p+1)(2p+3)} + \frac{b(2p+1)}{(2p+1)(2p+3)}$$
$$2 = a(2p+3) + b(2p+1)$$
$$2 = 2ap + 3a + 2bp + b$$
$$2 = (2a+2b)p + (3a+b)$$
$$\Rightarrow -\begin{cases} a+b=0 \\ 3a+b=2 \end{cases} \Rightarrow \begin{cases} a=1 \\ b=-1 \end{cases}$$
$$\Rightarrow \frac{2}{(2p+1)(2p+3)} = \frac{1}{2p+1} - \frac{1}{2p+3}$$

គេបាន

$$\sum_{p=0}^{n} \frac{2}{(2p+1)(2p+3)} = \sum_{p=0}^{n} \left(\frac{1}{2n+1} - \frac{1}{2p+3} \right)$$
$$= \left(1 - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{7}\right) + \cdots\cdots + \left(\frac{1}{2n+1} - \frac{1}{2n+3}\right)$$
$$= 1 - \frac{1}{2n+3} = \frac{2(n+1)}{2n+3}$$

ដូច្នេះ $S_n = \frac{2n+2}{2n+3}$

ខ. គណនាលីមីត

ដោយ $S_n = \dfrac{2n+2}{2n+3} \Rightarrow \lim\limits_{n\to+\infty} S_n = \lim\limits_{n\to+\infty} \dfrac{2n+2}{2n+3} = \lim\limits_{n\to+\infty} \dfrac{2+\dfrac{2}{n}}{2+\dfrac{3}{n}} = 1$

ដូច្នេះ $\lim\limits_{n\to+\infty} S_n = 1$

15. គេមានស៊េរីអនន្តមួយ $r^2 + \dfrac{r^2}{r^2+1} + \dfrac{r^2}{\left(r^2+1\right)} + \cdots + \dfrac{r^2}{\left(r^2+1\right)^n} + \cdots$

បង្ហាញថា ស៊េរីអនន្តនេះរួមចំពោះគ្រប់តម្លៃ r ។

បង្ហាញថា ស៊េរីអនន្តនេះរួម

យើងមាន $r^2 + \dfrac{r^2}{r^2+1} + \dfrac{r^2}{\left(r^2+1\right)^2} + \cdots + \dfrac{r^2}{\left(r^2+1\right)^n} + \cdots$ ជាស៊េរីធរណីមាត្រ មានតួទី1 ស្មើនឹង r^2

និង ផលធៀបស្មើនឹង $\dfrac{1}{r^2+1}$

-បើ $r=0$ នោះគេបាន $r^2 + \dfrac{r^2}{r^2+1} + \dfrac{r^2}{\left(r^2+1\right)^2} + \cdots + \dfrac{r^2}{\left(r^2+1\right)^n} + \cdots = 0$

-បើ $r \neq 0$ នោះគេបាន $0 < \dfrac{1}{r^2+1} < 1$

យើងបាន $r^2 + \dfrac{1}{r^2+1} + \dfrac{r^2}{\left(r^2+1\right)^2} + \cdots + \dfrac{r^2}{\left(r^2+1\right)^n} + \cdots = \dfrac{r^2}{1-\dfrac{1}{r^2+1}} = r^2+1$

ដូច្នេះ ស៊េរីនេះរួម ចំពោះគ្រប់តម្លៃ r ។

16. កំណត់ស៊េរីខាងក្រោម តើស៊េរីណារួម ស៊េរីណារីក? បើជាស៊េរីរួម ចូររកផលបូក។

ក. $\sum\limits_{n=1}^{\infty} 2\left(\cos\dfrac{\pi}{3}\right)^n$ ខ. $\sum\limits_{n=1}^{\infty} \left(\tan\dfrac{\pi}{4}\right)^n$

គ. $\sum\limits_{n=1}^{\infty} \dfrac{n}{2n+5}$ ឃ. $\sum\limits_{n=1}^{\infty} \dfrac{5(-1)^n}{4^n}$

ក. យើងមាន $\sum\limits_{n=1}^{\infty} 2\left(\cos\dfrac{\pi}{3}\right)^n$

ដោយ $\cos\dfrac{\pi}{3} = \dfrac{1}{2}$ នោះគេបាន $\sum\limits_{n=1}^{\infty} 2\left(\cos\dfrac{\pi}{3}\right) = \sum\limits_{n=1}^{\infty} 2\left(\dfrac{1}{2}\right)^n$ ជាស៊េរីធរណីមាត្រមានផលធៀបរួមស្មើ $\dfrac{1}{2}$

និង តួទីមួយស្មើនឹង 1

យើងបាន $S_n = \sum_{k=1}^{n} 2\left(\frac{1}{2}\right)^n = \frac{1}{1-\frac{1}{2}} = 2$

ដូច្នេះ ស៊េរី $\sum_{n=1}^{\infty} 2\left(\cos\frac{\pi}{3}\right)^n$ ជាស៊េរីរួម និងមាន ផលបូកស្មើ 2 ។

ខ. យើងមាន $\sum_{n=1}^{\infty}\left(\tan\frac{\pi}{4}\right)^n$

ដោយ $\tan\frac{\pi}{4} = 1$ នោះគេបាន $\sum_{n=1}^{\infty}\left(\tan\frac{\pi}{4}\right)^n = \sum_{n=1}^{\infty} 1^n = 1+1+1+\cdots$ ជាស៊េរីរីក

ដូច្នេះ $\sum_{n=1}^{\infty}\left(\tan\frac{\pi}{4}\right)^n$ ជាស៊េរីរីក។

គ. យើងមាន $\sum_{n=1}^{\infty}\frac{n}{2n+5}$

យើងបាន $a_n = \frac{n}{2n+5} \Rightarrow \lim_{n\to+\infty} a_n = \lim_{n\to+\infty}\frac{n}{2n+5} = \lim_{n\to+\infty}\frac{1}{2+\frac{5}{n}} = \frac{1}{2}$

ដូច្នេះ ស៊េរី $\sum_{n=1}^{\infty}\frac{n}{2n+5}$ ជាស៊េរីរីក។

ឃ. យើងមាន $\sum_{n=1}^{\infty}\frac{5(-1)^n}{4^n}$ ជាស៊េរីធរណីមាត្រ មានតួទីមួយគឺ $-\frac{5}{4}$ និងផលធៀបរួមស្មើ $-\frac{1}{4} \in (-1,1)$

យើងបាន ស៊េរីនេះជាស៊េរីរួមរក $\frac{-\frac{5}{4}}{1+\frac{1}{4}} = -1$

ដូច្នេះ ស៊េរី $\sum_{n=1}^{\infty}\frac{5(-1)^n}{4^n}$ ជាស៊េរីរួមរក -1 ហើយ ផលបូកនៃស៊េរីស្មើ -1។

17. គេមានត្រីកោណ ABC មួយមានផ្ទៃក្រឡាស្មើនឹង 6 ឯកតា។ សង់ត្រីកោណ $A'B'C'$ ដោយយក A', C' និង C' ជាចំណុចកណ្តាលនៃជ្រុងត្រីកោណ ABC ។ ចូរកំណត់ផលបូក $S = S_1 + S_2 + S_3 + \cdots$ ។

កំណត់ផលបូក $S = S_1 + S_2 + S_3 + \cdots$

ΔABC មានផ្ទៃក្រឡាស្មើ 6 ឯកតា ហើយ $\Delta A'B'C'$ ដែល $A'B'C'$ ជាចណុចកណ្តាលនៃជ្រុងត្រីកោណ ABC នោះយើងបាន

$$S_{\Delta ABC}=4S_{\Delta A'B'C'}=6$$
$$\Rightarrow S_{\Delta A'B'C'}=\frac{S_{\Delta ABC}}{4}=\frac{6}{4}$$

តាង $S_1=S_{\Delta ABC}\ , S_2=S_{\Delta A'B'C'}\ ,\cdots\cdots$

យើងបាន

$$\begin{aligned}&S_1=6\\&S_2=\frac{6}{4}\\&S_3=\frac{6}{4.4}=\frac{6}{4^2}\\(+)\ &S_4=\frac{6}{4.4.4}=\frac{6}{4^3}\\&\vdots\\&S_n=\frac{6}{4^{n-1}}\\&\vdots\end{aligned}$$

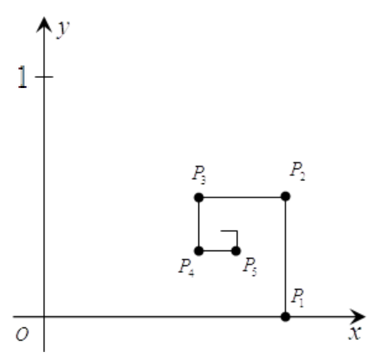
$$S_1+S_2+S_3+S_4+\cdots+S_n+\cdots=6+\frac{6}{4}+\frac{6}{4^2}+\frac{6}{4^3}+\cdots+\frac{6}{4^n}+\cdots$$

យើងបាន $S=6+\frac{6}{4}+\frac{6}{4^2}+\frac{6}{4^3}+\cdots+\frac{6}{4^{n-1}}+\cdots$ ជាស៊េរីធរណីមាត្រមានផលធៀបរួមស្មើ $\frac{1}{4}$ និងតួទី1 ស្មើ 6

នាំឱ្យ $S=\dfrac{6}{1-\frac{1}{4}}=8$

ដូច្នេះ $S=8$ឯកតាក្រឡាផ្ទៃ។

18. នៅលើប្លង់កូអរដោនេគេមានចំណុច P មួយរំកិលពីគល់ $O(0,0)$ ទៅចំណុច P_1 នៅលើអ័ក្ស (ox) 1ឯកតា រួចរកិលឡើងលើស្របនឹង អ័ក្ស (oy) $\frac{1}{2}$ ឯកតា នៃ OP_1 បន្ទាប់មករំកិលមកក្រោយតាមទិសអវិជ្ជមានស្រប នឹងអ័ក្ស (ox) $\left(\frac{1}{2}\right)^2$ ឯកតានៃ P_1P_2 ហើយរំកិលបន្តបន្ទាប់ទៀតច្រើនដង។ រកកូអរដោនេចំណុច P នៅទីតាំងចុងក្រោយ។



រកកូអរដោនេចំណុច P នៅទីតាំងចុងក្រោយ

តាង x_{2n-1} ជាកូអរដោនេនៃចំណុច P បន្ទាប់ពីផ្លាស់ទីបាន $2n-1$ ដង និង y_{2n} ជាកូអរដោនេនៃចំណុច P បន្ទាប់ពីផ្លាស់ទីបាន $2n$ ដង

យើងបាន

$$x_{2n-1}=1-\frac{1}{2^2}+\frac{1}{2^4}-\frac{1}{2^6}+\cdots+\left(-\frac{1}{2^2}\right)^{n-1}$$

$$y_{2n}=\frac{1}{2}-\frac{1}{2^3}+\frac{1}{2^5}-\frac{1}{2^7}+\cdots+\frac{1}{2}\left(-\frac{1}{2^2}\right)^{n-1}$$

យើងបាន ស៊េរីទាំងពីរនេះ ជាស៊េរីធរណីមាត្រ មានផលធៀបរួមស្មើ $-\frac{1}{2^2}$

យើងបាន

$$\lim_{n\to+\infty} x_{2n-1}=\frac{1}{1-\left(-\frac{1}{2^2}\right)}=\frac{4}{5}$$

$$\lim_{n\to+\infty} y_{2n}=\frac{\frac{1}{2}}{1-\left(-\frac{1}{2^2}\right)}=\frac{2}{5}$$

ដូច្នេះ បន្ទាប់ពីផ្លាស់ទីជាច្រើនដង P មានកូអរដោនេ $\left(\frac{4}{5},\frac{2}{5}\right)$ ។