មេរៀនទី១: អាំងតេក្រាលកំណត់

ដំណោះស្រាយលំហាត់

1. ដោយប្រើនិយមន័យ គណនាអាំងតេក្រាលខាងក្រោម:

ក. $\int_0^2 3xdx$

គេមាន:

ចែកអង្កត់ $[0,2]$ ជា n ស្មើៗគ្នា គេបាន:

$x_i = \frac{2i}{n}$ និង $\Delta x_i = x_i - x_{i-1} = \frac{2i}{n} - \frac{2(i-1)}{n} = \frac{2}{n}$ ដែល $i = 0;1;2;.......$

ជ្រើសរើស $\xi_i \in [x_{i-1}; x_i]$ ដែល $\xi_i = x_i = \frac{2i}{n}$

$\Rightarrow f(\xi_i) = 3\xi_i = 3\frac{2i}{n}$

ផលបូកអាំងតេក្រាល

$$S_n = \sum_{i=1}^{n} f(\xi_i)\Delta x_i = \sum_{i=1}^{n}\left[\left(3\frac{2i}{n}\right)\left(\frac{2}{n}\right)\right] = \frac{12}{n^2}\sum_{i=1}^{n} i = \frac{12}{n^2}\times\frac{n(n+1)}{2}$$

លីមីតនៃ S_n កាលណា $n\to\infty$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(\frac{12}{n^2}\times\frac{n(n+1)}{2}\right) = 6$$

ខ. $\int_2^4 4xdx$

គេមាន:

ចែកអង្កត់ $[2,4]$ ជា n ស្មើៗគ្នា គេបាន:

$x_i = 2 + \frac{2i}{n}$ និង $\Delta x_i = x_i - x_{i-1} = \left(2+\frac{2i}{n}\right) - \left(2+\frac{2(i-1)}{n}\right) = \frac{2}{n}$ ដែល $i = 0;1;2;.......$

ជ្រើសរើស $\xi_i \in [x_{i-1}; x_i]$ ដែល $\xi_i = x_i = 2 + \frac{2i}{n}$

$\Rightarrow f(\xi_i) = 4\xi_i = 4\left(2+\frac{2i}{n}\right) = 8 + \frac{8i}{n}$

ផលបូកអាំងតេក្រាល

$$S_n=\sum_{i=1}^{n} f\left(\xi_i\right)\Delta x_i=\sum_{i=1}^{n}\left[\left(8+\frac{8i}{n}\right)\left(\frac{2}{n}\right)\right]=\sum_{i=1}^{n}\left(\frac{16}{n}+\frac{16i}{n^2}\right)=\frac{16n}{n}+\frac{16\left[\frac{n(n+1)}{2}\right]}{n^2}$$

$$=\frac{16n}{n}+\frac{8n(n+1)}{n^2}$$

លីមីតនៃ S_n កាលណា $n\to\infty$

$$\lim_{n\to\infty} S_n=\lim_{n\to\infty}\left(\frac{16n}{n}+\frac{8n(n+1)}{n^2}\right)=24$$

គ. $\int_0^2 x^2dx$

គេមាន:

ចែកអង្កត់ $[0,2]$ ជា n ស្មើៗគ្នា គេបាន:

$$x_i=\frac{2i}{n} \text{ និង } \Delta x_i=x_i-x_{i-1}=\frac{2i}{n}-\frac{2(i-1)}{n}=\frac{2}{n} \text{ ដែល } i=0;1;2;.......$$

ជ្រើសរើស $\xi_i\in[x_{i-1};x_i]$ ដែល $\xi_i=x_i=\frac{2i}{n}$

$$\Rightarrow f\left(\xi_i\right)=\xi^2{}_i=\left(\frac{2i}{n}\right)^2$$

ផលបូកអាំងតេក្រាល

$$S_n=\sum_{i=1}^{n} f\left(\xi_i\right)\Delta x_i=\sum_{i=1}^{n}\left[\left(\frac{2i}{n}\right)^2\left(\frac{2}{n}\right)\right]=\frac{8}{n^3}\sum_{i=1}^{n} i^2=\frac{8}{n^3}\times\frac{n(n+1)(2n+1)}{6}$$

$$=\frac{8\left[n(n+1)(2n+1)\right]}{6n^3}$$

លីមីតនៃ S_n កាលណា $n\to\infty$

$$\lim_{n\to\infty} S_n=\lim_{n\to\infty}\left(\frac{8\left[n(n+1)(2n+1)\right]}{6n^3}\right)=\frac{8}{3}$$

ឃ. $\int_0^2 (x^2-5x)dx$

គេមាន:

ចែកអង្កត់ $[0,2]$ ជា n ស្មើៗគ្នា គេបាន:

$$x_i=\frac{2i}{n} \text{ និង } \Delta x_i=x_i-x_{i-1}=\frac{2i}{n}-\frac{2(i-1)}{n}=\frac{2}{n} \text{ ដែល } i=0;1;2;.......$$

ជ្រើសរើស $\xi_i \in [x_{i-1}; x_i]$ ដែល $\xi_i = x_i = \frac{2i}{n}$

$$\Rightarrow f(\xi_i) = \xi_i^2 - 5\xi_i = \left(\frac{2i}{n}\right)^2 - \frac{5\times 2i}{n} = \frac{4i^2}{n^2} - \frac{10i}{n}$$

ផលបូកអាំងតេក្រាល

$$S_n = \sum_{i=1}^{n} f(\xi_i)\Delta x_i = \sum_{i=1}^{n}\left[\left(\frac{4i^2}{n^2} - \frac{10i}{n}\right)\left(\frac{2}{n}\right)\right] = \frac{8}{n^3}\sum_{i=1}^{n} i^2 - \frac{20}{n^2}\sum_{i=1}^{n} i$$

$$= \frac{8\left[\frac{n(n+1)(2n+1)}{6}\right]}{n^3} - \frac{20n(n+1)}{2n^2} = \frac{4n(n+1)(2n+1)}{3} - \frac{20n(n+1)}{2n^2}$$

លីមីតនៃ S_n កាលណា $n \to \infty$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(\frac{4n(n+1)(2n+1)}{3} - \frac{20n(n+1)}{2n^2}\right) = \frac{8}{3} - 10 = -\frac{22}{3}$$

ង. $\int_1^2 x^2 dx$

គេមាន:

ចែកអង្កត់ $[1,2]$ ជា n ស្មើៗគ្នា គេបាន:

$$x_i = 1 + \frac{i}{n} \text{ និង } \Delta x_i = x_i - x_{i-1} = 1 + \frac{i}{n} - \left(1 + \frac{i-1}{n}\right) = \frac{1}{n} \quad \text{ដែល } i = 0;1;2;.......$$

ជ្រើសរើស $\xi_i \in [x_{i-1}; x_i]$ ដែល $\xi_i = x_i = 1 + \frac{i}{n}$

$$\Rightarrow f(\xi_i) = \xi_i^2 = \left(1 + \frac{i}{n}\right)^2 = 1 + \frac{2i}{n} + \left(\frac{i}{n}\right)^2$$

ផលបូកអាំងតេក្រាល

$$S_n = \sum_{i=1}^{n} f(\xi_i)\Delta x_i = \sum_{i=1}^{n}\left\{\left[1 + \frac{2i}{n} + \left(\frac{i}{n}\right)^2\right]\left(\frac{1}{n}\right)\right\} = \sum_{i=1}^{n}\left(\frac{1}{n} + \frac{2i}{n^2} + \frac{i^2}{n^3}\right)$$

$$= 1 + \frac{2}{n^2}\sum_{i=1}^{n} i + \frac{1}{n^3}\sum_{i=1}^{n} i^2 = 1 + \left(\frac{2}{n^2} \times \frac{n(n+1)}{2}\right) + \left(\frac{1}{n^3} \times \frac{n(n+1)(2n+1)}{6}\right)$$

$$= 1 + \frac{n(n+1)}{n^2} + \frac{n(n+1)(2n+1)}{6n^3}$$

លីមីតនៃ S_n កាលណា $n \to \infty$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(1 + \frac{n(n+1)}{n^2} + \frac{n(n+1)(2n+1)}{6n^3}\right) = 1 + 1 + \frac{1}{3} = \frac{7}{6}$$

2. គណនាតម្លៃប្រហែលនៃផ្ទៃក្រឡាដែលខណ្ឌដោយក្រាបតាង $y=f(x)$ និងអ័ក្ស $x'ox$ លើចន្លោះ $[a,b]$ ដែល:

ក. $f(x)=9-x^2$ ដែល $[a,b]=[-3,2], n=5$

ដោយគេចែកចន្លោះ $[a,b]=[-3,2]$ ជា 5 ចន្លោះស្មើគ្នា គេបាន:

$$x_0=-3, x_1=-2, x_2=-1, x_3=0, x_4=1, x_5=2$$

ប្រវែងនៃចន្លោះតូចនីមួយៗគឺ: $\Delta x=\frac{2-(-3)}{5}=\frac{5}{5}=1$

រើសតំលៃ c_1,c_2,c_3,c_4,c_5 ដែល:

$$c_1=\frac{-3+(-2)}{2}=-\frac{5}{2}\ ,\ c_2=\frac{-2+(-1)}{2}=-\frac{3}{2}\ ,\ c_3=\frac{-1+0}{2}=-\frac{1}{2}\ ,\ c_4=\frac{0+1}{2}=\frac{1}{2}\ ,c_5=\frac{1+2}{2}=\frac{3}{2}$$

; (ដែល $c_i=\frac{x_{i-1}+x_i}{2}$)

គេបាន:

$$S_5=f(c_1)\Delta x+f(c_2)\Delta x+f(c_3)\Delta x+f(c_4)\Delta x+f(c_5)\Delta x$$

$$=f\left(-\frac{5}{2}\right)\Delta x+f\left(-\frac{3}{2}\right)\Delta x+f\left(-\frac{1}{2}\right)\Delta x+f\left(\frac{1}{2}\right)\Delta x+f\left(\frac{3}{2}\right)\Delta x$$

$$=\left[9-\left(-\frac{5}{2}\right)^2\right]\times1+\left[9-\left(-\frac{3}{2}\right)^2\right]\times1+\left[9-\left(-\frac{1}{2}\right)^2\right]\times1+\left[9-\left(\frac{1}{2}\right)^2\right]\times1$$

$$+\left[9-\left(\frac{3}{2}\right)^2\right]\times1=\frac{11}{4}+\frac{27}{4}+\frac{35}{4}+\frac{35}{4}+\frac{27}{4}=\frac{11+27+35+35+27}{4}=\frac{135}{4}$$

ខ. $f(x)=\frac{1}{x+2}$ ដែល $[a,b]=[-1,3], n=4$

- ដោយគេចែកចន្លោះ $[a,b]=[-1,3]$ ជា 4 ចន្លោះស្មើគ្នា គេបាន:

$$x_0=-1, x_1=0, x_2=1, x_3=2, x_4=3$$

- ប្រវែងនៃចន្លោះតូចនីមួយៗគឺ: $\Delta x=\frac{3-(-1)}{4}=\frac{4}{4}=1$
- ជ្រើសរើសតំលៃ c_1,c_2,c_3,c_4 ដែល:

$$c_1=\frac{-1+0}{2}=-\frac{1}{2}\ ,\ c_2=\frac{0+1}{2}=\frac{1}{2}\ ,\ c_3=\frac{1+2}{2}=\frac{3}{2}\ ,\ c_4=\frac{2+3}{2}=\frac{5}{2}$$

គេបាន:

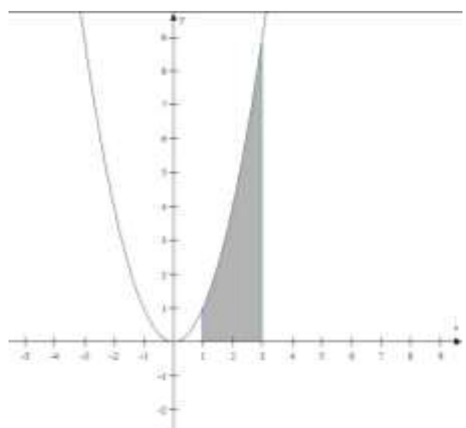
$$S_4 = f(c_1)\Delta x + f(c_2)\Delta x + f(c_3)\Delta x + f(c_4)\Delta x$$
$$= f\left(-\frac{1}{2}\right)\Delta x + f\left(\frac{1}{2}\right)\Delta x + f\left(\frac{3}{2}\right)\Delta x + f\left(\frac{5}{2}\right)\Delta x$$
$$= \left(\frac{1}{-\frac{1}{2}+2}\right)\times 1 + \left(\frac{1}{\frac{1}{2}+2}\right)\times 1 + \left(\frac{1}{\frac{3}{2}+2}\right)\times 1 + \left(\frac{1}{\frac{5}{2}+2}\right)\times 1$$
$$= \frac{2}{3}+\frac{2}{5}+\frac{2}{7}+\frac{2}{9} = \frac{(2\times 105)+(2\times 63)+(2\times 45)+(2\times 35)}{315}$$
$$= \frac{210+126+90+70}{315} = \frac{496}{315} = 1.5746$$

3. គណនាផ្ទៃក្រឡាខណ្ឌដោយក្រាបតាងអនុគមន៍ និង អ័ក្ស $x'ox$ លើចន្លោះ ដែលឲ្យ:

ក. $f(x) = x^2$ ដែល $x \in [1,3]$

គេបាន:

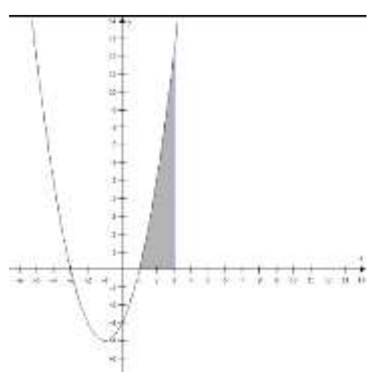
$$S = \int_1^3 f(x)dx = \int_1^3 x^2 dx$$
$$= \left[\frac{1}{3}x^3\right]_1^3 = \frac{1}{3}\left(3^3 - 1^3\right)$$
$$= \frac{26}{3}$$



ខ. $f(x) = x^2 + 2x - 3$ ដែល $x \in [1,3]$

គេបាន:

$$S = \int_1^3 \left(x^2 + 2x - 3\right)dx = \left[\frac{1}{3}x^3 + x^2 - 3x\right]_1^3$$
$$= \frac{1}{3}\left(3^3 - 1^3\right) + \left(3^2 - 1^2\right) - 3(3-1)$$
$$= \frac{26}{3} + 8 - 6 = \frac{32}{3}$$



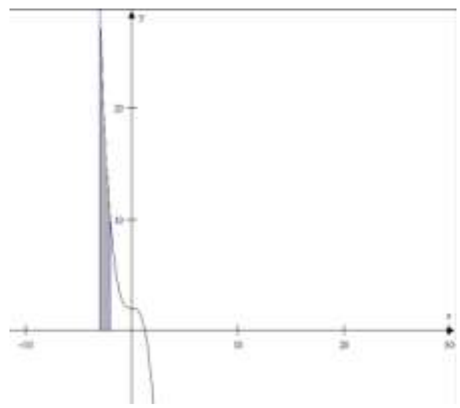
គ. $f(x) = 2 - x^3$ ដែល $x \in [-3, -2]$

គេបាន:

$$S = \int_{-3}^{-2} f(x)dx = \int_{-3}^{-2}(2 - x^3)dx = \left[2x - \frac{1}{4}x^4\right]_{-3}^{-2}$$

$$= 2[-2-(-3)] - \frac{1}{4}\left[(-2)^4 - (-3)^4\right]$$

$$= 2 - \frac{1}{4}(16 - 81) = \frac{8+65}{4} = \frac{73}{4}$$

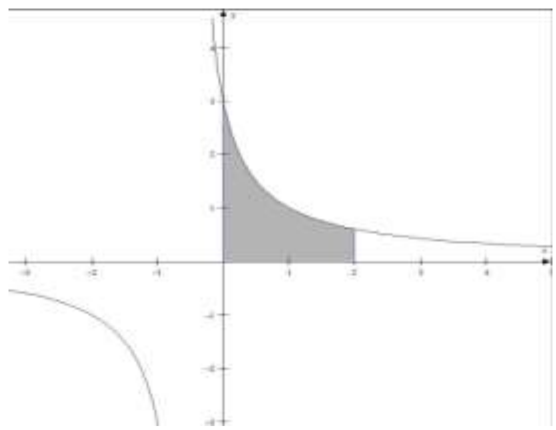


ឃ. $f(x) = \dfrac{3}{2x+1}$ ដែល $x \in [0, 2]$

គេបាន:

$$S = \int_0^2 f(x)dx = \int_0^2 \left(\frac{3}{2x+1}\right)dx$$

$$= 3\left[\frac{1}{2}\ln|2x+1|\right]_0^2 = \frac{3}{2}\ln 5$$



4. គណនាក្រឡាផ្ទៃដែលខណ្ឌដោយក្រាបតាងអនុគមន៍ទាំងពីរ:

ក. $f(x) = x^2 + 2$ និង $g(x) = x$, $x \in [2, 5]$

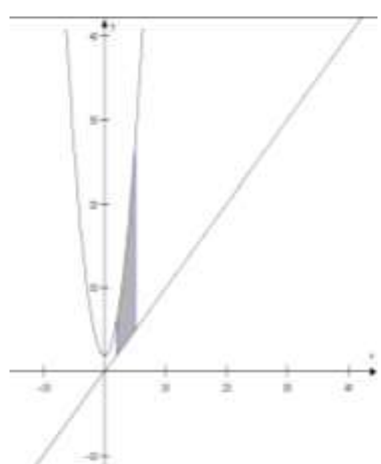
គេបាន:

$$S = \int_2^5 [f(x) - g(x)]dx = \int_2^5 [(x^2+2) - x]dx$$

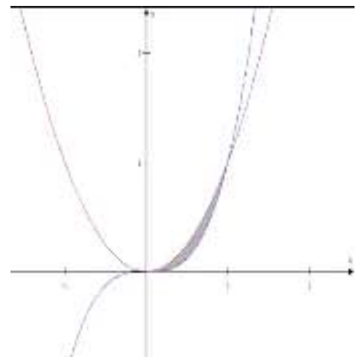
$$= \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x\right]_2^5$$

$$= \frac{1}{3}(5^3 - 2^3) - \frac{1}{2}(5^2 - 2^2) + 2(5-2)$$

$$= \frac{117}{3} - \frac{21}{2} + 6 = \frac{234 - 63 + 36}{6} = \frac{207}{6} = \frac{69}{2}$$



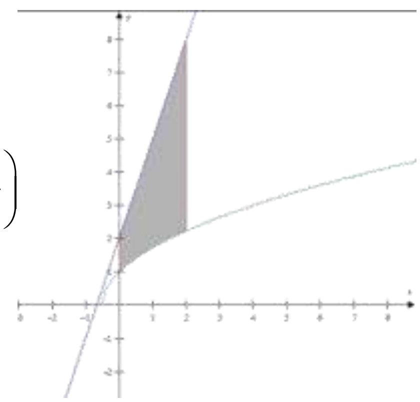
ខ. $f(x)=x^2$ និង $g(x)=x^3, x\in[0,1]$



គេបាន:

$$S=\int_0^1\left[f(x)-g(x)\right]dx=\int_0^1\left[x^2-x^3\right]dx$$
$$=\left[\frac{1}{3}x^3-\frac{1}{4}x^4\right]_0^1=\frac{1}{3}\left(1^3-0^3\right)-\frac{1}{4}\left(1^4-0^4\right)$$
$$=\frac{1}{3}-\frac{1}{4}=\frac{1}{12}$$

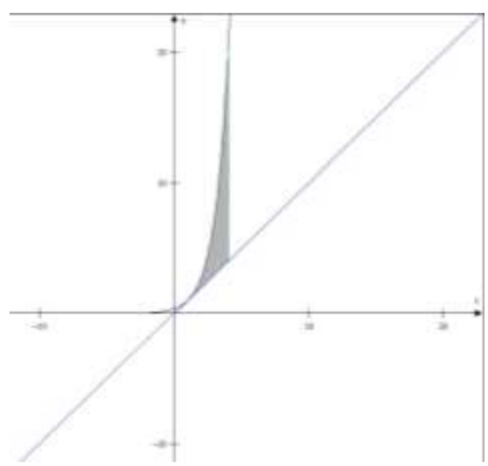
គ. $f(x)=\sqrt{2x+1}$ និង $g(x)=3x+2,\ x\in[0,2]$



គេបាន:

$$S=\int_0^2\left[g(x)-f(x)\right]dx=\int_0^2\left[(3x+2)-\left(\sqrt{2x+1}\right)\right]dx$$
$$=\left[\frac{3}{2}x^2+2x-\frac{1}{3}(2x+1)^{\frac{3}{2}}\right]_0^2=\left(10-\frac{\sqrt{125}}{3}\right)-\left(0-\frac{1}{3}\right)$$
$$=\frac{31-5\sqrt{5}}{3}$$

ឃ. $f(x)=e^{x-1}$ និង $g(x)=x,\ x\in[1,4]$



គេមាន:

$$S=4\int_1^4\left[f(x)-g(x)\right]dx=\int_1^4\left(e^{x-1}-x\right)dx$$
$$=\left[e^{x-1}-\frac{1}{2}x^2\right]_1^4=\left(e^3-8\right)-\left(1-\frac{1}{2}\right)$$
$$=e^3-\frac{17}{2}$$

5. គណនាក្រលាផ្ទៃខណ្ឌដោយខ្សែកោងតាងអនុគមន៍ $x=y^2$ និង $y=x-2$

គេមាន:

$y=x-2$ នាំអោយ $x=y+2$

ម្យ៉ាងទៀត:

$y^2 = y + 2$

$y^2 - y - 2 = 0$

$y_1 = -1, y_2 = 2$

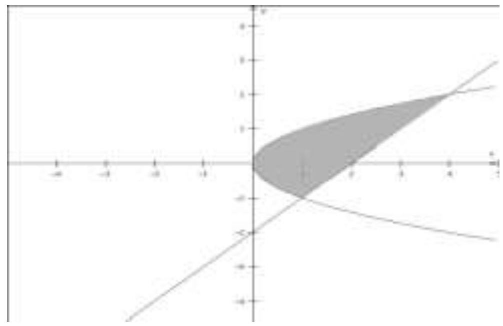
គេបាន:

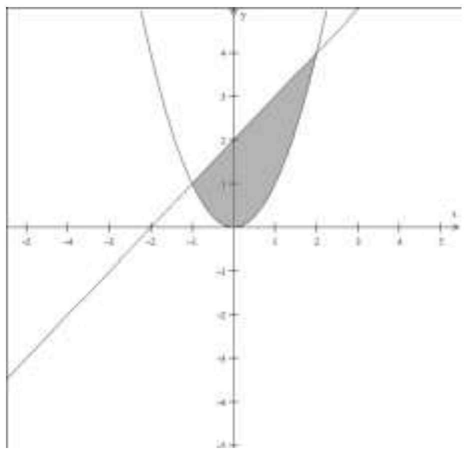
$$S = \int_{-1}^{2}\left(y + 2 - y^2\right)dy$$

$$= \left[\frac{1}{2}y^2 + 2y - \frac{1}{3}y^3\right]_{-1}^{2}$$

$$= \left(6 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$$

$$= \frac{20 - 3 + 12 - 2}{6} = \frac{9}{2}$$





7. គណនាក្រលាផ្ទៃខណ្ឌដោយខ្សែកោងតាងអនុគមន៍ $y = f(x) = \frac{1}{x+1}$ និង $y = g(x) = e^{0.7x}$ និង

$x \in [0, 4]$

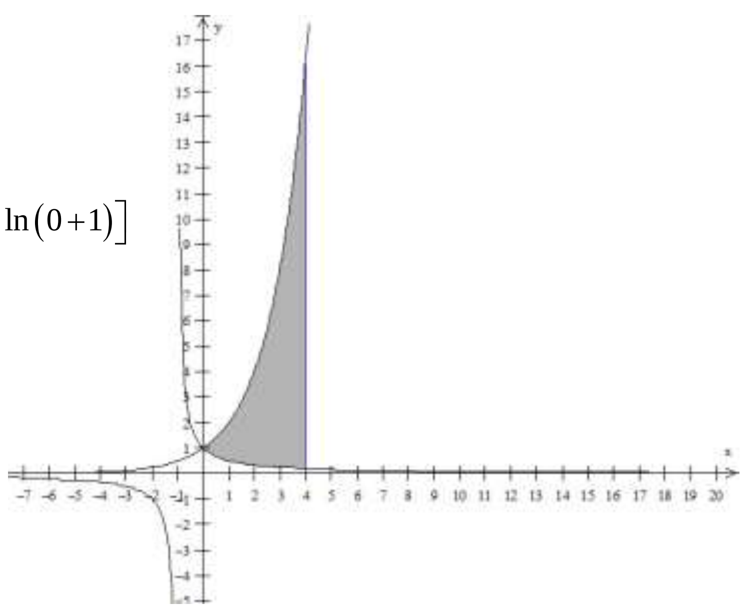
គេមាន:

$$S = \int_{0}^{4}\left(e^{0.7x} - \frac{1}{x+1}\right)dx$$

$$= \left[\frac{1}{0.7}e^{0.7x} - \ln(x+1)\right]_{0}^{4}$$

$$= \left[\frac{1}{0.7}\left(e^{0.7\times 4} - e^{0.7\times 0}\right)\right] - \left[\ln(4+1) - \ln(0+1)\right]$$

$$= 20.45$$



មេរៀនទី២: មាឌសូលីត និងប្រវែងធ្នូ

ដំណោះស្រាយលំហាត់

១. *ក.* គណនាមាឌនៃសូលីតបរិវត្តកំណត់បានពីរង្វិលជុំវិញអ័ក្ស $x'ox$ នៃផ្ទៃក្រឡា ដែលខណ្ឌដោយក្រាប តាងអនុគមន៍ $y=2x+1$ អ័ក្ស $x'ox$ បន្ទាត់ឈរ $x=1$ និង $x=3$ ។

ខ. គណនាមាឌនៃសូលីតបរិវត្តកំណត់បានពីរង្វិលជុំវិញអ័ក្ស $x'ox$ នៃផ្ទៃក្រឡា ដែលខណ្ឌដោយក្រាប តាងអនុគមន៍ $y=x^2+1$ អ័ក្ស $x'ox$ បន្ទាត់ឈរ $x=0$ និង $x=3$ ។

គ. គណនាមាឌនៃសូលីតបរិវត្តកំណត់បានពីរង្វិលជុំវិញអ័ក្ស $x'ox$ នៃផ្ទៃក្រឡា ដែលខណ្ឌដោយក្រាប តាងអនុគមន៍ $y=\sqrt{x}-3$ អ័ក្ស $x'ox$ បន្ទាត់ឈរ $x=4$ និង $x=9$ ។

ដំណោះស្រាយ

ក. $V=\pi\int_1^3(2x+1)^2dx=\pi\int_1^3(4x^2+4x+1)dx$

$$=\pi\left(\frac{4x^3}{3}+2x^2+x\right)\Big|_1^3=\pi\left[(4.9+2.9+3)-\left(\frac{4}{3}+2+1\right)\right]$$

$$=\pi\left(57-\frac{13}{3}\right)=\pi\frac{158}{3}$$

ខ. $=\pi\left(\frac{x^5}{5}+\frac{2x^3}{3}+x\right)\Big|_0^3=\pi\left[\left(\frac{3^5}{5}+\frac{54}{3}+3\right)\right]=\pi\left(\frac{243}{5}+21\right)=\pi\frac{348}{5}$

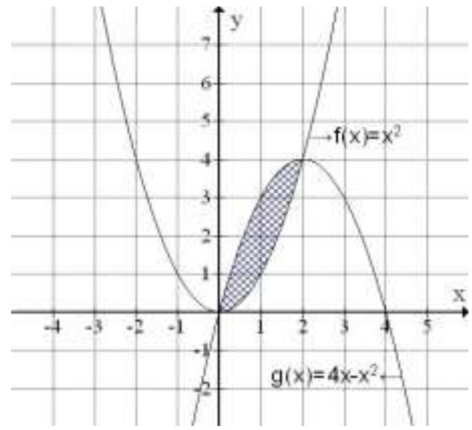
គ. $V=\pi\int_4^9(\sqrt{x}-3)dx=\pi\int_4^9(x-6\sqrt{x}+9)dx=\pi\left(\frac{x^2}{2}-4\sqrt{x^3}+9x\right)\Big|_4^9$

$$=\pi\left[\left(\frac{81}{2}-4.27+81\right)-(8-32+36)\right]=\pi\left(\frac{81}{2}-39\right)$$

២. *ក.* គណនាមាឌនៃសូលីតបរិវត្តកំណត់បានពីរង្វិលជុំវិញអ័ក្ស $x'ox$ នៃផ្ទៃក្រឡា ដែលខណ្ឌដោយក្រាបតាង អនុគមន៍ $f(x)=x^2$ និង $g(x)=4x-x^2$ ។

ខ. គណនាមាឌនៃសូលីតបរិវត្តកំណត់បានពីរង្វិលជុំវិញអ័ក្ស $x'ox$ នៃ ផ្ទៃក្រឡា ដែលខណ្ឌដោយក្រាបតាង អនុគមន៍ $f(x)=x^2-2x+3$ និង $g(x)=9-x$ ។

ដំណោះស្រាយ



ក. $V=\pi\int_0^2\left[g^2(x)-f^2(x)\right]dx$

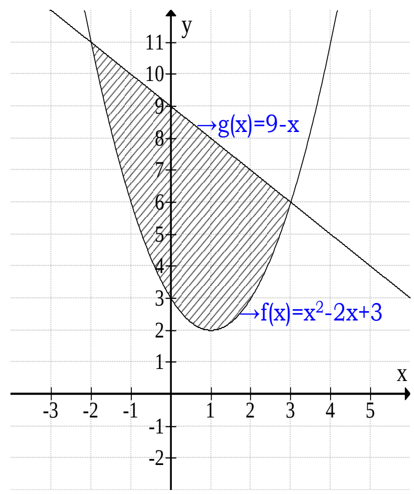
ព្រោះ $f(x)\le g(x)$ លើចន្លោះ $[0,2]$

$$\Rightarrow V=\pi\int_0^2\left[\left(4x-x^2\right)^2-x^4\right]dx$$

$$=\pi\int_0^2\left(16x^2-8x^3\right)dx$$

$$=8\pi\int_0^2\left(2x^2-x^3\right)dx$$

$$=8\pi\left(\frac{2}{3}x^3-\frac{x^4}{4}\right)\Big|_0^2=8\pi\left(\frac{16}{3}-4\right)=\frac{32}{3}\pi$$



ខ. ចំណុចប្រសព្វរវាងខ្សែកោងទាំងពីរគឺ:

$x^2-2x+3=9-x$ នោះយើងបាន

$x=3$ និង $x=-2$

ចំពោះគ្រប់ $x\in[-2,3]$ គេបាន $g(x)\ge f(x)$

$$\Rightarrow V=\pi\int_{-2}^3\left[g^2(x)-f^2(x)\right]dx$$

$$=\pi\int_{-2}^3\left[\left(9-x^2\right)^2-\left(x^2-2x+3\right)^2\right]dx$$

$$=\pi\int_{-2}^3\left(-x^4+4x^3-9x^2-6x+72\right)dx$$

$$=\pi\left(-\frac{x^5}{5}+x^4-3x^3-3x^2+72x\right)\Big|_{-2}^3$$

៣. ចំពោះអនុគមន៍ $F(x)=\int_1^x\sin\pi t\,dt$ ។ គណនា:

ក. $F(-1)$ ខ. $F'(0)$ គ. $F'\left(\frac{1}{2}\right)$ ឃ. $F''(x)$ ។

ដំណោះស្រាយ

ក. យើងមាន $F(x)=\int_1^x\sin\pi t\,dt=\left(-\frac{\cos\pi t}{\pi}\right)\Big|_1^x$

$$=\frac{1}{\pi}\left(\cos\pi-\cos\pi x\right)$$

$$\Rightarrow F(-1)=\frac{1}{\pi}\left(\cos\pi-\cos\pi\right)=0$$

ខ. តាមទ្រឹស្តីបទយើងមាន $\frac{d}{dx}F(x)=\frac{d}{dx}\left[\int_a^x f(t)dt\right]=f(x)$

យើងបាន $F(x)=\int_1^x \sin(\pi t)dt \Leftrightarrow \frac{d}{dx}F(x)=\frac{d}{dx}\left[\int_1^x \sin(\pi t)dt\right]$

$$\Rightarrow F'(x)=\sin(\pi x)$$

$$\Rightarrow F'(0)=\sin(\pi .0)=0$$

ដូចនេះ $F'(0)=0$

គ. $F'\left(\frac{1}{2}\right)$

$$F'(x)=\sin(\pi x)\Rightarrow F'\left(\frac{1}{2}\right)=\sin\left(\frac{\pi}{2}\right)=1$$

ឃ. $F''(x)$

$$F'(x)=\sin(\pi x)$$

$$F''(x)=\left[\sin(\pi x)\right]'=\pi\cos(\pi x)$$

៤. ផ្ទៃក្រឡា A ខណ្ឌដោយក្រាបតាងអនុគមន៍ $g(t)=4-\frac{4}{t^2}$ និង អ័ក្សអាប់ស៊ីស លើចន្លោះ $[1,x]$ កំណត់ដោយ $A(x)=\int_1^x\left(4-\frac{4}{t^2}\right)dt$

ក. គណនា A ជាអនុគមន៍នៃ x ។ តើក្រាបតាងអនុគមន៍ A មានសមីការអាស៊ីតូតដេកឬទេ?

ខ. រកសមីការអាស៊ីមតូតទ្រេតនៃក្រាបតាងអនុគមន៍ A ។

ដំណោះស្រាយ

ក. គណនា A ជាអនុគមន៍នៃ x

$$A(x)=\int_1^x\left(4-\frac{4}{t^2}\right)dt \quad =\left(4t+\frac{4}{t}\right)\Big|_1^x=\left(4x+\frac{4}{x}\right)-(4+4)$$

$$=4x+\frac{4}{x}-8=\frac{4x^2-8x+4}{x}$$

យើងមាន $\lim_{x\to\pm\infty}A(x)=\lim_{x\to\pm\infty}\left(\frac{4x^2-8x+4}{x}\right)=\pm\infty$

ខ.រកសមីការអាស៊ីមតូតទ្រេតនៃក្រាបតាងអនុគមន៍ A

យើងមាន $A(x)=\dfrac{4x^2-8x+4}{x}=4x-8+\dfrac{4}{x}$

តែ $\lim\limits_{x\to\infty}\dfrac{4}{x}=0\Rightarrow g(x)=4x-8$ ជាអាស៊ីមតូតទ្រេតនៃ $A(x)$

៥. គណនា $F'(x)$ បើ

ក. $F(x)=\int_x^{x+2}(4t+1)dt$

ខ. $F(x)=\int_{-x}^{x}t^3dt$

គ. $F(x)=\int_0^{\sin x}\sqrt{t}dt$

ឃ. $F(x)=\int_2^{x^2}\dfrac{1}{t^2}dt$

ង. $F(x)=\int_0^{x^3}\sin tdt$

ច. $F(x)=\int_{\sin x}^{0}\dfrac{1}{2+t}dt$

ដំណោះស្រាយ

ក. យើងមាន $F(x)=\int_x^{x+2}(4t+1)dt$

$$=\left(2t^2+t\right)\Big|_x^{x+2}$$

$$=2(x+2)^2+(x+2)-(2x^2+x)$$

$$=\cancel{2}x^2+8x+8+\cancel{x}+2-\cancel{2}x^2-\cancel{x}$$

$$=8x+10\quad\Rightarrow F'(x)=8$$

ខ.យើងមាន $F(x)=\int_{-x}^{x}t^3dt=\dfrac{t^4}{4}\Big|_{-x}^{x}=\dfrac{x^4}{4}-\dfrac{x^4}{4}=0$

គ. យើងមាន $F(x)=\int_0^{\sin x}\sqrt{t}dt=\dfrac{2}{3}\sqrt{t^3}\Big|_0^{\sin x}=\dfrac{2}{3}\left(\sqrt{\sin^3 x}\right)$

$$\Rightarrow F'(x)=\frac{\cancel{2}}{3}\cdot\frac{(\sin^3 x)'}{\cancel{2}\sqrt{\sin^3 x}}=\frac{\cancel{3}\cos x\sin^2 x}{\cancel{3}\sqrt{\sin^3 x}}=\cos x\sqrt{\sin x}$$

ឃ. យើងមាន $F(x)=\int_2^{x^2}\dfrac{1}{t^2}dt=\left(-\dfrac{1}{t}\right)\Big|_2^{x^2}=\left(-\dfrac{1}{x^2}+\dfrac{1}{2}\right)$

$$\Rightarrow F'(x)=\left(-\frac{1}{x^2}+\frac{1}{2}\right)'=\frac{2}{x^3}$$

ង. យើងមាន $F(x)=\int_0^{x^3}\sin tdt$

$$\Rightarrow F(x)=(-\cos t)\Big|_0^{x^3}=1-\cos x^3$$

$$\Rightarrow F'(x)=\left(1-\cos x^3\right)'=3x^2\sin x^3$$

៥. យើងមាន $F(x)=\int_{\sin x}^{0}\frac{1}{2+t}dt \quad =\int_{\sin x}^{0}\frac{(2+t)'}{2+t}dt=\ln(2+t)\Big|_{\sin x}^{0}$

$$=\ln 2-\ln(2+\sin x)$$

$$\Rightarrow F'(x)=\left[\ln 2-\ln(2+\sin x)\right]'=\frac{(2+\sin x)'}{(2+\sin x)}=\frac{\cos x}{2+\sin x}$$

៦. បង្ហាញថាបើ $f(x)$ ជាអនុគមន៍សេស នោះ $F(x)=\int_a^x f(t)dt$ គ្រប់ $x\in R$ ជាអនុគមន៍សេសដែរឬទេ?

ដំណោះស្រាយ

យើងមាន $F(x)=\int_a^x f(t)dt$ និង $f(t)$ ជាអនុគមន៍សេស

$$\Rightarrow F(-x)=\int_a^{-x} f(t)dt$$

តាង $u=-t\Rightarrow du=-dt$ បើ $t=a\Rightarrow u=-a \quad ,t=-x\Rightarrow u=x$

$$\Rightarrow F(-x)=-\int_{-a}^{x} f(-u)du$$

$$=-\int_{-a}^{x} f(-t)dt$$

$$=-\int_{-a}^{x}\left(-f(t)\right)dt$$ ព្រោះ $f(t)$ ជាអនុគមន៍សេស

$$F(-x)=\int_{-a}^{x} f(t)dt$$

បើ $a=0$ នោះ $F(-x)=F(x)=\int_0^x f(t)dt$

$\Rightarrow F(x)$ មិនមែនជាអនុគមន៍សេសទេ។

បើ $a\neq 0$ ក៏មិនសេស

៧. Admin មិនសូវយល់ប្រធាន

៨. ក. គណនាប្រវែងធ្នូនៃក្រាបតាងអនុគមន៍ $y=\frac{x^3}{3}+\frac{1}{4x}$ ពី $x=1$ ទៅ $x=3$ ។

ខ. ឧបមាថា $f(x)=\frac{1}{2}(e^x+e^{-x})$ ។ បន្ទាត់ជួបអ័ក្សអរដោនេត្រង់ចំនុច B ហើយប៉ះនឹងក្រាបតាង f ត្រង់ចំណុច $A(a,f(a))$ ដែល $a>0$ ។ ប្រៀបធៀបប្រវែងអង្កត់ AB និងប្រវែងធ្នូនៃក្រាបតាង f នៅចន្លោះបន្ទាត់ឈរ $x=0$ និង $x=a$ ។

ដំណោះស្រាយ

ក. ប្រវែងធ្នូតាងអនុគមន៍ $y=\frac{x^3}{3}+\frac{1}{4x}$ ពី $x=1$ ទៅ $x=3$

$$\Rightarrow y'=x^2-\frac{1}{4x^2}$$

ប្រវែងធ្នូនៃក្រាបតាងអនុគមន៍កំណត់ដោយ

$$L=\int_1^3\sqrt{1+\left(x^2-\frac{1}{4x^2}\right)^2}\,dx$$

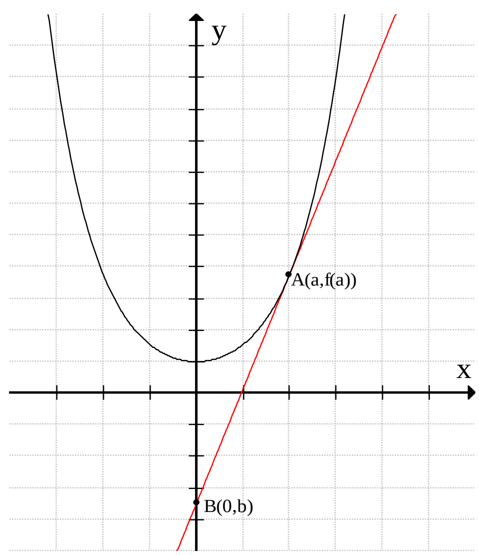
$$=\int_1^3\sqrt{1+x^4-\frac{1}{2}+\frac{1}{16x^4}}\,dx$$

$$=\int_1^3\sqrt{\left(x^2+\frac{1}{4x^2}\right)^2}\,dx$$

$$=\int_1^3\left(x^2+\frac{1}{4x^2}\right)dx$$

$$=\left(\frac{x^3}{3}-\frac{1}{4x}\right)\Bigg|_1^3=\left(9-\frac{1}{12}-\frac{1}{3}+\frac{1}{4}\right)=\frac{26}{3}$$

ខ. ប្រៀបធៀបប្រវែងអង្កត់ AB និងប្រវែងធ្នូនៃក្រាបតាង f នៅចន្លោះបន្ទាត់ឈរ $x=0$ និង $x=a$ ។



ប្រវែងនៃខ្សែកោងលើចន្លោះ$[0,a]$

$$L=\int_0^a\sqrt{1+[f'(x)]^2}\,dx$$ ដែល $f'(x)=\frac{1}{2}\left(e^x-e^{-x}\right)$

$$=\int_0^a\sqrt{1+\frac{1}{4}\left(e^x-e^{-x}\right)^2}\,dx$$

$$=\int_0^a\sqrt{1+\frac{1}{4}\left(e^{2x}-2+e^{-2x}\right)}dx$$

$$=\int_0^a\sqrt{\frac{1}{2}+\frac{1}{4}(e^{2x}+e^{-2x})}dx$$

$$=\int_0^a\sqrt{\frac{1}{4}\left(e^x+e^{-x}\right)^2}\,dx$$

$$=\int_0^a\frac{1}{2}\left(e^x+e^{-x}\right)dx$$

$$=\frac{1}{2}\left(e^x-e^{-x}\right)\Big|_0^a=\frac{1}{2}\left(e^a-e^{-a}-1+1\right)=\frac{1}{2}\left(e^a-e^{-a}\right)\qquad(1)$$

ប្រវែងអង្កត់ AB

$$AB=\sqrt{a^2+\left(b-f(a)\right)^2}$$

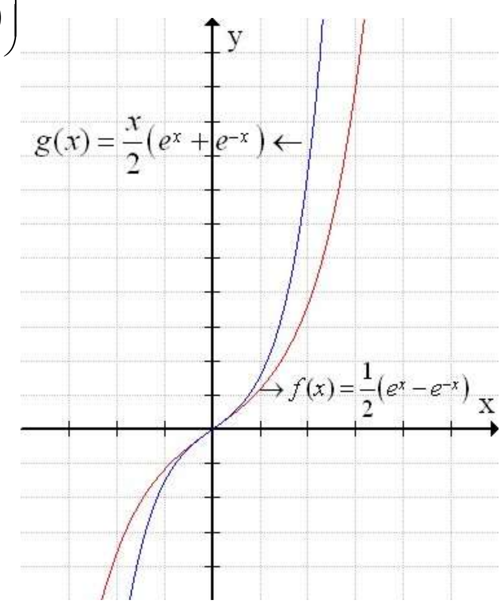
យើងមាន $y=mx+b$ ជាសមីការបន្ទាត់ប៉ះនៃ f ត្រង់ a

$$\Rightarrow y=f'(a)(x-a)+f(a)$$
$$=\frac{1}{2}\left(e^a-e^{-a}\right)(x-a)+\frac{1}{2}\left(e^a+e^{-a}\right)$$

ម្យ៉ាងទៀត $y=mx+b$ កាត់អ័ក្សអាប់ស៊ីសត្រង់ B

មានន័យថាពេល $x=0 \quad ,y=b=-\frac{a}{2}\left(e^a-e^{-a}\right)+\frac{1}{2}\left(e^a+e^{-a}\right)$

$$\Rightarrow AB=\sqrt{a^2+\left(-\frac{a}{2}\left(e^a-e^{-a}\right)+\frac{1}{2}\left(e^a+e^{-a}\right)-\frac{1}{2}\left(e^a+e^{-a}\right)\right)^2}$$
$$=\sqrt{a^2+\frac{a^2}{4}\left(e^{2a}-2+e^{-2a}\right)}$$
$$=|a|\sqrt{1-\frac{1}{2}+\frac{1}{4}\left(e^{2a}+e^{-2a}\right)}$$
$$=a\sqrt{\frac{1}{4}\left(e^a+e^{-a}\right)^2} \quad \text{ព្រោះ } a>0$$
$$=\frac{a}{2}\left(e^a+e^{-a}\right) \qquad (2)$$



តាមរូបយើងបាន $\frac{a}{2}\left(e^a+e^{-a}\right)\geq\frac{1}{2}\left(e^a-e^{-a}\right)$ គ្រប់ $a>0$

ដំណោះស្រាយលំហាត់ជំពូក

១. គណនាអាំងតេក្រាលខាងក្រោមនេះ

ក. $\int_{-2}^{2}\left(4-x^2\right)\left(2+x\right)^n dx$ ខ. $\int_6^8 \frac{x}{x^2-6x+8}dx$

គ. $\int_0^1 \frac{x^2}{x^2-x-2}dx$ ឃ. $\int_1^3 \frac{x-1}{\sqrt{x}}dx$

ង. $\int_0^{\pi} \cos mx \cos nx dx \quad \left(m,n\in\mathbb{N}\right)$ ច. $\int_0^1 \frac{x}{1+3x^2}dx$

ឆ. $\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{3+\cos^2 x}dx$ ជ. $\int_0^{\frac{\pi}{2}} x^2\cos^2 xdx$

ដំណោះស្រាយ

ក. $I=\int_{-2}^{2}\left(4-x^2\right)\left(2+x\right)^n dx=\int_{-2}^{2}\left(2-x\right)\left(2+x\right)^{n+1}dx$

តាង $u=2-x\Rightarrow du=-dx$

$$dv=\left(2+x\right)^{n+1}dx\Rightarrow v=\int\left(2+x\right)^{n+1}dx=\frac{\left(2+x\right)^{n+2}}{n+2}$$

$$\Rightarrow I=\left(2-x\right)\cdot\left.\frac{\left(2+x\right)^{n+2}}{n+2}\right|_{-2}^{2}+\int_{-2}^{2}\frac{\left(2+x\right)^{n+2}}{n+2}dx$$

$$=0+\int_{-2}^{2}\frac{\left(2+x\right)^{n+2}}{n+2}dx$$

$$=\left.\frac{\left(2+x\right)^{n+3}}{\left(n+2\right)\left(n+3\right)}\right|_{-2}^{2}=\frac{4^{n+3}}{\left(n+2\right)\left(n+3\right)}$$

ខ. $J=\int_6^8 \frac{x}{x^2-6x+8}dx$

យើងមាន $x^2-6x+8=\left(x-2\right)\left(x-4\right)$

$$\Rightarrow \frac{x}{x^2-6x+8}=\frac{A}{x-2}+\frac{B}{x-4}=\frac{Ax-4A+Bx-2B}{\left(x-2\right)\left(x-4\right)}$$

យើងបានប្រព័ន្ធសមីការ

$$\begin{cases}A+B=1\\-4A-2B=0\end{cases}\Rightarrow\begin{cases}A+B=1\\2A+B=0\end{cases}\Rightarrow\begin{cases}A=-1\\B=2\end{cases}$$

នោះ $\int_6^8 \frac{x}{x^2-6x+8}dx = \int_6^8\left[\frac{-1}{x-2}+\frac{2}{x-4}\right]dx$

$= 2\int_6^8 \frac{1}{x-4}dx - \int_6^8 \frac{1}{x-2}dx$

$= 2\ln(x-4)\Big|_6^8 - \ln(x-2)\Big|_6^8$

$= 2(\ln 4 - \ln 2) - (\ln 6 - \ln 4)$

$= \ln\frac{8}{3} = 3\ln 2 - \ln 3$

គ. $K = \int_0^1 \frac{x^2}{x^2-x-2}dx$

យើងមាន $x^2 - x - 2 = (x+1)(x-2)$

$\Rightarrow \frac{x^2}{x^2-x-2} = \frac{x^2}{(x+1)(x-2)} = \frac{A}{x+1}+\frac{B}{x-2}$

យើងបានប្រព័ន្ធសមីការ

$\begin{cases} A+B=1 \\ -2A+B=2 \end{cases} \Rightarrow A=\frac{-1}{3} \ , \ B=\frac{4}{3}$

$\Rightarrow K = \frac{4}{3}\int_0^1\frac{dx}{x-2} - \frac{1}{3}\int_0^1\frac{dx}{x+1}$

$= \frac{4}{3}\int_0^1\frac{dx}{2-x} - \frac{1}{3}\int_0^1\frac{dx}{x+1}$

$= \frac{4}{3}\ln(2-x)\Big|_0^1 - \frac{1}{3}\ln(x+1)\Big|_0^1$

$= \frac{4}{3}(-\ln 2) - \frac{1}{3}\ln 2$

$= -\frac{5}{3}\ln 2$

ឃ. $L = \int_1^3 \frac{x-1}{\sqrt{x}}dx = \int_1^3\left(\frac{x}{\sqrt{x}} - \frac{1}{\sqrt{x}}\right)dx = \int_1^3\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)dx$

$= \left(\frac{2}{3}\sqrt{x^3} - 2\sqrt{x}\right)\Bigg|_1^3 = \left(2\sqrt{3}-2\sqrt{3}\right) - \left(\frac{2}{3}-2\right) = \frac{4}{3}$

ង. $M = \int_0^\pi \cos mx \cos nx dx \quad (m,n\in\mathbb{N})$

យើងមាន $\cos(mx)\cos(nx) = \frac{1}{2}\left[\cos(mx+nx)+\cos(mx-nx)\right]$

$$\Rightarrow M=\frac{1}{2}\int_0^{\pi}\left(\cos\left(mx+nx\right)+\cos\left(mx-nx\right)\right)dx$$
$$=\frac{1}{2}\left(\frac{\sin\left(mx+nx\right)}{m+n}+\frac{\sin\left(mx-nx\right)}{m-n}\right)\Bigg|_0^{\pi}$$
$$=\frac{1}{2}\left[\left(\frac{\sin\left(\pi\left(m+n\right)\right)}{m+n}+\frac{\sin\left(\pi\left(m-n\right)\right)}{m-n}\right)-0\right]$$
$$=\frac{1}{2}\left[\frac{\sin k\pi}{k}+\frac{\sin k'\pi}{k'}\right]$$ ព្រោះ $m,n\in\mathbb{N}\Rightarrow\begin{cases}m+n=k\in\mathbb{N}\\ m-n=k'\in\mathbb{N}\end{cases}$
$$\Rightarrow M=0$$

ច. $N=\int_0^1\frac{x}{1+3x^2}dx$

តាង $u=3x^2+1 \Rightarrow du=6xdx$

$$\Rightarrow dx=\frac{du}{6x}\ ,\begin{cases}x=0\\ x=1\end{cases}\Rightarrow\begin{cases}u=1\\ u=4\end{cases}$$
$$\Rightarrow N=\frac{1}{6}\int_1^4\frac{du}{u}=\frac{1}{6}\ln u\Big|_1^4=\frac{1}{6}\left(\ln 4\right)=\frac{\ln 2}{3}$$

ឆ. $P=\int_0^{\frac{\pi}{2}}\frac{\sin 2x}{3+\cos^2 x}dx=\int_0^{\frac{\pi}{2}}\frac{2\sin x\cos x}{3+\cos^2 x}dx$

តាង $u=3+\cos^2 x\Rightarrow du=-2\cos x\sin xdx$

$$\Rightarrow dx=-\frac{du}{2\cos x\sin x}\ ,\begin{cases}x=0\\ x=\frac{\pi}{2}\end{cases}\Rightarrow\begin{cases}u=4\\ u=3\end{cases}$$
$$\Rightarrow P=\int_4^3\frac{2\sin x\cos x}{u}\cdot\left(-\frac{du}{2\sin x\cos x}\right)$$
$$=-\int_4^3\frac{du}{u}=\int_3^4\frac{du}{u}=\ln u\Big|_3^4=\ln 4-\ln 3$$
$$=2\ln 2-\ln 3$$

ជ. $Q=\int_0^{\frac{\pi}{2}}x^2\cos^2 xdx$

តាង $u=x^2\Rightarrow du=2xdx$

$$dv=\cos^2 x=\frac{1+\cos 2x}{2}\Rightarrow v=\int\frac{1+\cos 2x}{2}dx$$
$$=\frac{1}{2}\left(x+\frac{\sin 2x}{2}\right)$$

$$\Rightarrow Q=\frac{x^2}{2}\left(x+\frac{\sin 2x}{2}\right)\Bigg|_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\left(x+\frac{\sin 2x}{2}\right)\frac{1}{2}\cdot 2xdx$$

$$=\frac{\pi^2}{8}\left(\frac{\pi}{2}+0\right)-\int_0^{\frac{\pi}{2}}\left(x^2+\frac{x\sin 2x}{2}\right)dx$$

$$=\frac{\pi^3}{16}-\frac{x^3}{3}\Bigg|_0^{\frac{\pi}{2}}-\frac{1}{2}\int_0^{\frac{\pi}{2}}x\sin 2xdx$$

$$=\frac{\pi^3}{48}-\frac{1}{2}\int_0^{\frac{\pi}{2}}x\sin xdx\quad(*)$$

តាង $\begin{cases}u=x\Rightarrow du=dx\\ dv=\sin 2x\Rightarrow v=\int\sin 2xdx=-\dfrac{\cos 2x}{2}\end{cases}$

$$\Rightarrow\int_0^{\frac{\pi}{2}}x\sin 2xdx=-\frac{x\cos 2x}{2}\Bigg|_0^{\frac{\pi}{2}}+\int_0^{\frac{\pi}{2}}\cos 2xdx$$

$$=\frac{\pi}{4}-\left(\frac{\sin 2x}{4}\right)\Bigg|_0^{\frac{\pi}{2}}=\frac{\pi}{4}$$

$$\Rightarrow(*)=\frac{\pi^3}{48}-\frac{1}{2}\cdot\frac{\pi}{4}=\frac{\pi\left(\pi^2-6\right)}{48}$$

២.គណនា

ក. $\int_0^{\pi}e^{-at}\cos^2\frac{t}{2}dt\quad(a\neq 0)$

ខ. $\int_1^{e}\frac{\sin(\pi\ln x)}{x}dx$

គ. $\int_0^{\sqrt{\ln 6}}\frac{x}{e^{-x^2}+e^{x^2}+2}dx$

ឃ. $\int_0^1\left(4-x^2-\sqrt{1-x^2}\right)dx$

ង. $\int_0^1\frac{dx}{\sqrt{4-x^2}}dx$

ច. $\int_0^a\sqrt{2ax-x^2}dx$

ឆ. $\int_0^a\left(x^2+a^2\right)^{\frac{3}{2}}dx\quad(a>0)$

ជ. $\int_0^1\frac{2x}{x^2-x+1}dx$

ដំណោះស្រាយ

ក. $\int_0^{\pi}e^{-at}\cos^2\frac{t}{2}dt\quad(a\neq 0)$

តាង $I=\int_0^{\pi}e^{-at}\cos^2\frac{t}{2}dt\quad(a\neq 0)$ និងមាន $\cos^2\frac{t}{2}=\frac{1+\cos t}{2}$

$$\Rightarrow I=\int_0^{\pi}e^{-at}\left(\frac{1+\cos t}{2}\right)dt$$

$$=\frac{1}{2}\int_0^{\pi}\left(e^{-at}+e^{-at}\cos t\right)dt$$

$$=\frac{1}{2}\left[\int_0^{\pi}e^{-at}dt+\int_0^{\pi}e^{-at}\cos tdt\right]$$

$$=\frac{1}{2}\left[-\frac{e^{-at}}{a}+I'\right]_0^{\pi}$$ តាង $I'=\int e^{-at}\cos tdt$

គណនា I' , តាង $u=\cos t\Rightarrow du=-\sin tdt$

$dv=e^{-at}dt\Rightarrow v=\int e^{-at}dt=-\frac{1}{a}\cdot e^{-at}$

$$\Rightarrow I'=-\frac{e^{-at}\cos t}{a}-\frac{1}{a}\int e^{-at}\sin tdt$$

គណនា $\int e^{-at}\sin tdt$

តាង $u=\sin t\Rightarrow du=\cos tdt$

$$dv=e^{-at}\Rightarrow v=\int e^{-at}dt=-\frac{e^{-at}}{a}$$

$$\Rightarrow\int e^{-at}\sin tdt=-\frac{e^{-at}\sin t}{a}+\frac{1}{a}\int e^{-at}\cos tdt$$

$$=-\frac{e^{-at}\sin t}{a}+\frac{I'}{a}$$

$$\Rightarrow I'=-\frac{e^{-at}\cos t}{a}-\frac{1}{a}\left(-\frac{e^{-at}\sin t}{a}+\frac{I'}{a}\right)$$

$$\Rightarrow I'\left(1+\frac{1}{a^2}\right)=\frac{e^{-at}\sin t}{a^2}-\frac{e^{-at}\cos t}{a}$$

$$\Rightarrow I'=\frac{ae^{-at}}{\left(a^2+1\right)}\left(\frac{\sin t}{a}-\cos t\right)$$

$$\Rightarrow I=\frac{1}{2}\left[-\frac{e^{-at}}{a}+\frac{ae^{-at}}{\left(a^2+1\right)}\left(\frac{\sin t}{a}-\cos t\right)\right]_0^{\pi}$$

$$\Rightarrow I=\frac{1}{2}\left[\frac{a}{e^{at}\left(a^2+1\right)}\left(\frac{\sin t}{a}-\cos t\right)-\frac{1}{ae^{at}}\right]_0^{\pi}$$

$$\Rightarrow I=\frac{1}{2}\left[\left(\frac{a}{e^{\pi a}\left(a^2+1\right)}-\frac{1}{ae^{\pi a}}\right)-\left(-\frac{a}{\left(a^2+1\right)}-\frac{1}{a}\right)\right]$$

$$\Rightarrow I=\frac{2a^2e^{\pi a}+e^{\pi a}-1}{2ae^{\pi a}\left(a^2+1\right)}$$

ខ. $J=\int_1^e \frac{\sin(\pi \ln x)}{x}dx$

តាង $u=\pi\ln x \Rightarrow du=\frac{\pi}{x}dx \Rightarrow dx=\frac{xdu}{\pi}$

$$\Rightarrow \int_1^e \frac{\sin(\pi\ln x)}{x}dx=\int_1^e \frac{\sin u}{x}\cdot\frac{xdu}{\pi}$$

$$=\frac{1}{\pi}\int_1^e \sin u du$$

$$=-\frac{1}{\pi}\cos u\Big|_1^e$$

$$=-\frac{1}{\pi}\cos(\pi\ln x)\Big|_1^e$$

$$=-\frac{1}{\pi}(\cos\pi-\cos 0)$$

$$=\frac{2}{\pi}$$

គ. $K=\int_0^{\sqrt{\ln 6}}\frac{x}{e^{-x^2}+e^{x^2}+2}dx$

គុណអង្គទាំងពីរនឹង e^{x^2} យើងបាន

$$K=\int_0^{\sqrt{\ln 6}}\frac{xe^{x^2}}{1+e^{2x^2}+2e^{x^2}}dx$$

$$=\frac{1}{2}\int_0^{\sqrt{\ln 6}}\frac{2xe^{x^2}}{\left(e^{x^2}+1\right)^2}dx$$

តាង $u=e^{x^2}+1 \Rightarrow du=2xe^{x^2}dx$

បើ $x=0\Rightarrow u=2$, $x=\sqrt{\ln 6}\Rightarrow u=e^{\ln 6}+1=7$

នោះ $K=\frac{1}{2}\int_2^7\frac{du}{u^2}=\frac{1}{2}\left(-\frac{1}{u}\right)\Big|_2^7=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{7}\right)=\frac{5}{28}$

ឃ. $F(x)=\int\frac{P_n(x)dx}{\sqrt{ax^2+bx+c}}$ គេអាចបម្លែងទៅជា

$$F(x)=Q_n(x)\sqrt{ax^2+bx+c}+\lambda\int\frac{dx}{\sqrt{ax^2+bx+c}}$$

$$\Rightarrow F'(x)=\left[Q_n(x)\sqrt{ax^2+bx+c}\right]'+\frac{\lambda}{\sqrt{ax^2+bx+c}}$$

ដែល $P_n(x)$ ជាពហុធាដឺក្រេលំដាប់ n

$\deg Q_n=\deg P_n-1$

ឃ. $L=\int_0^1\left(4-x^2-\sqrt{1-x^2}\right)dx$

$$=\int_0^1\left(4-x^2\right)dx-\int_0^1\sqrt{1-x^2}\,dx$$

$$=\left(4x-\frac{x^3}{3}\right)\Big|_0^1-\int_0^1\sqrt{1-x^2}\,dx$$

$$=\frac{11}{3}-\int_0^1\frac{1-x^2}{\sqrt{1-x^2}}dx\ (*)$$

គណនា $\int_0^1\frac{1-x^2}{\sqrt{1-x^2}}dx$

$$=\int_0^1\frac{dx}{\sqrt{1-x^2}}-\int_0^1\frac{x^2}{\sqrt{1-x^2}}dx\quad(**)$$

តែ $F(x)=\int\frac{x^2}{\sqrt{1-x^2}}dx$ អាចសរសេរជារាង

$$F(x)=ax\sqrt{1-x^2}+\lambda\int\frac{dx}{\sqrt{1-x^2}}$$

$$\Rightarrow F'(x)=a\sqrt{1-x^2}-\frac{ax^2}{\sqrt{1-x^2}}+\frac{\lambda}{\sqrt{1-x^2}}$$

$$=\frac{a\left(1-x^2\right)-ax^2+\lambda}{\sqrt{1-x^2}}$$

$$\Rightarrow x^2=-2ax^2+a+\lambda$$

$$\Rightarrow\begin{cases}-2a=1\\a+\lambda=0\end{cases}\Rightarrow a=-\frac{1}{2}\ ,\ \lambda=\frac{1}{2}$$

$$\Rightarrow\int\frac{x^2dx}{\sqrt{1-x^2}}=\left(-\frac{1}{2}x\sqrt{1-x^2}\right)+\frac{1}{2}\int\frac{dx}{\sqrt{1-x^2}}$$

ជំនួសចូលក្នុង$(**)$

$$\Rightarrow\int_0^1\frac{x^2}{\sqrt{1-x^2}}dx=\frac{1}{2}\left[Arc\sin x+x\sqrt{1-x^2}\right]_0^1=\frac{\pi}{4}$$

ជំនួសតម្លៃ $\frac{\pi}{4}$ ចូលក្នុង $(*)$ យើងបាន $L=\frac{11}{3}-\frac{\pi}{4}$

ង. $M=\int_0^1\frac{dx}{\sqrt{4-x^2}}dx$

តាមរូបមន្តយើងបាន

$$\int_0^1 \frac{dx}{\sqrt{4-x^2}} = Arc\sin\left(\frac{x}{2}\right)\Big|_0^1 = Arc\sin\frac{1}{2} - Arc\sin 0 = \frac{\pi}{6}$$

ច. $F(x) = \int_0^a \sqrt{2ax-x^2}\,dx$

យើងមាន $F(x) = \int_0^a \sqrt{2ax-x^2}\,dx$

$= \int_0^1 \frac{2ax-x^2}{\sqrt{2ax-x^2}}\,dx$ យើងអាចបម្លែងជា

$$F(x) = (b_0x+b_1)\sqrt{2ax-x^2} + \lambda\int \frac{dx}{\sqrt{2ax-x^2}}$$

$$\Rightarrow F'(x) = b_0\sqrt{2ax-x^2} + \frac{(b_0x+b_1)(a-x)}{\sqrt{2ax-x^2}} + \frac{\lambda}{\sqrt{2ax-x^2}}$$

$$\Rightarrow 2ax-x^2 = b_0(2ax-x^2) + (b_0x+b_1)(a-x) + \lambda$$

យើងបាន $\begin{cases} 2b_0 = 1 \\ 3ab_0 - b_1 = 2a \\ ab_1 + \lambda = 0 \end{cases} \Rightarrow b_0 = \frac{1}{2}\ ,\ b_1 = -\frac{a}{2}\ ,\ \lambda = \frac{a^2}{2}$

$$\Rightarrow F(x) = \left[\left(\frac{x-a}{x}\right)\sqrt{2ax-x^2}\right]_0^a + \frac{a^2}{2}\int_0^a \frac{dx}{\sqrt{2ax-x^2}}$$

តែ $\int_0^a \frac{dx}{\sqrt{2ax-x^2}} = \int_0^a \frac{dx}{\sqrt{a^2-(a-x)^2}} = Arc\sin\frac{a-x}{a}\Big|_0^a$

$$\Rightarrow F(x) = \left[\left(\frac{x-a}{2}\right)\sqrt{2ax-x^2} + \frac{a^2}{2}Arc\sin\frac{a-x}{a}\right]_0^a$$

$$= -\frac{a^2}{2}Arc\sin 1 = -\frac{a^2}{2}\cdot\frac{\pi}{2}$$

ឆ. $F(x) = \int_0^a (x^2+a^2)^{\frac{3}{2}}\,dx$

$$= \int_0^a \frac{(x^2+a^2)^2}{\sqrt{x^2+a^2}}\,dx$$

$$=\int_0^a \frac{x^4+2a^2x^2+a^4}{\sqrt{x^2+a^2}}dx$$ តាមរូបមន្តយើងបាន

$$F(x)=\left(b_0x^3+b_1x^2+b_2x+b_3\right)\sqrt{x^2+a^2}+\lambda\int\frac{dx}{\sqrt{x^2+a^2}}$$

$$F'(x)=\left(3b_0x^2+2b_1x+b_2\right)\sqrt{x^2+a^2}+\frac{x\left(b_0x^3+b_1x^2+b_2x+b_3\right)}{\sqrt{x^2+a^2}}+\frac{\lambda}{\sqrt{x^2+a^2}}$$

$$=\frac{\left(3b_0x^2+2b_1x+b_2\right)\left(x^2+a^2\right)+x\left(b_0x^3+b_1x^2+b_2x+b_3\right)+\lambda}{\sqrt{x^2+a^2}}\ (*)$$

$$\Rightarrow x^4+2a^2x^2+a^4=(*)$$

ប្រៀបធៀបមេគុណយើងបាន:

$$\begin{cases}4b_0=1\\3b_1=0\\3b_0a^2+2b_2=2a^2\\2b_1a^2+b_3=0\\b_2a^2+\lambda=a^4\end{cases}\Rightarrow\begin{cases}b_0=\frac{1}{4}\quad ,b_1=0\\b_2=\frac{5a^2}{8}\quad ,b_3=0\\\lambda=\frac{3a^4}{8}\end{cases}$$

$$\Rightarrow F(x)=\left[\left(\frac{x^3}{4}+\frac{5a^2x}{8}\right)\sqrt{x^2+a^2}+\frac{3a^4}{8}\int\frac{dx}{\sqrt{x^2+a^2}}\right]_0^a$$

$$\Rightarrow F(x)=\left[\left(\frac{x^3}{4}+\frac{5a^2x}{8}\right)\sqrt{x^2+a^2}+\frac{3a^4}{8}\ln\left|x+\sqrt{x^2+a^2}\right|\right]_0^a$$

$$\Rightarrow F(x)=\frac{7\sqrt{2}a^4+3a^4\ln\left(1+\sqrt{2}\right)}{8}\quad ,a>0$$

ជ. $F(x)=\int_0^1\frac{2x}{x^2-x+1}dx$

$$=\int_0^1\frac{2x-1}{x^2-x+1}dx+\int_0^1\frac{dx}{x^2-x+1}$$

តាង $u=x^2-x+1\Rightarrow du=(2x-1)dx$

និង $x^2-x+1=\left(x-\frac{1}{2}\right)^2+\frac{3}{4}$

$$\Rightarrow F(x)=\int_0^1\frac{(x^2-x+1)'}{(x^2-x+1)}dx+\int_0^1\frac{dx}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$\Rightarrow F(x)=\left[\ln\left|x^2-x+1\right|+\frac{1}{\frac{\sqrt{3}}{2}}Arctg\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right]_0^1$$

$$\Rightarrow F(x)=\left[\ln\left|x^2-x+1\right|+\frac{2}{\sqrt{3}}Arctg\frac{2x-1}{\sqrt{3}}\right]_0^1$$

$$=\left[\ln(1-1+1)+\frac{2}{\sqrt{3}}Arctg\left(\frac{1}{\sqrt{3}}\right)\right]-\left[\ln 1+\frac{2}{\sqrt{3}}Arctg\left(-\frac{1}{\sqrt{3}}\right)\right]$$

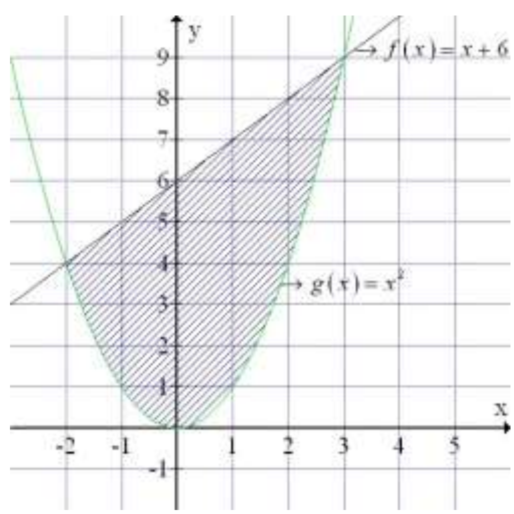
$$=\frac{2}{\sqrt{3}}Arctg\frac{1}{\sqrt{3}}-\frac{2}{\sqrt{3}}Arctg\left(-\frac{1}{\sqrt{3}}\right)$$

$$=\frac{2}{\sqrt{3}}\cdot\frac{\pi}{6}-\frac{2}{\sqrt{3}}\cdot\left(-\frac{\pi}{6}\right)$$

៣.គណនាមាឌសូលីតបរិវត្តកំណត់បានពីរង្វិលជុំវិញអ័ក្ស $x'ox$ នៃ ផ្ទៃដែលខណ្ឌដោយក្រាភិចតាងអនុគមន៍ $f(x)=x+6$ និង $g(x)=x^2$ លើចន្លោះ $x\in[-2,3]$ ។

ដំណោះស្រាយ

យើងពិនិត្យឃើញថា $f(x)\ge g(x)$ លើចន្លោះ $[-2,3]$



$$V=\pi\int_{-2}^{3}(x+6)^2dx-\pi\int_{-2}^{3}x^4dx$$

$$=\pi\int_{-2}^{3}\left(x^2+12x+36-x^4\right)dx$$

$$=\pi\left[\frac{x^3}{3}+6x^2+36x-\frac{x^5}{5}\right]_{-2}^{3}$$

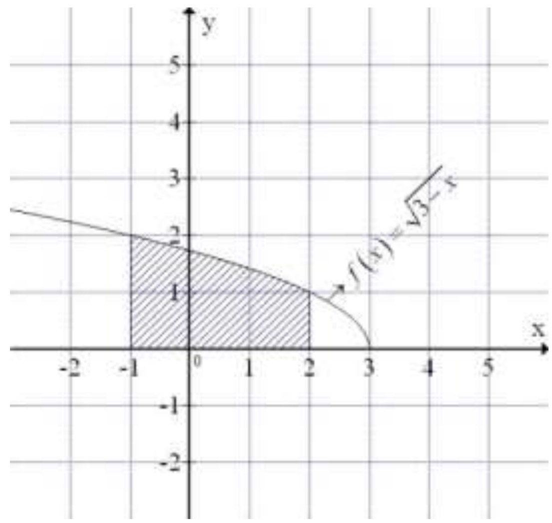
$$=\pi\left(164+\frac{8}{3}\right)=\frac{500\pi}{3}$$

៤.គណនាមាឌសូលីតបរិវត្តកំណត់បានពីរង្វិលនៃផ្ទៃខណ្ឌ

ដោយក្រាភិចតាងអនុគមន៍ $y=\sqrt{3-x}$ និងអ័ក្ស $x'ox$

$(-1\le x\le 2)$ ជុំវិញអ័ក្ស $x'ox$ ។

ដំណោះស្រាយ



គណនាមាឌសូលីត

យើងបាន:

$V=\pi\int_{-1}^{2}\left(\sqrt{3-x}\right)^2 dx$

$=\pi\int_{-1}^{2}(3-x)dx$

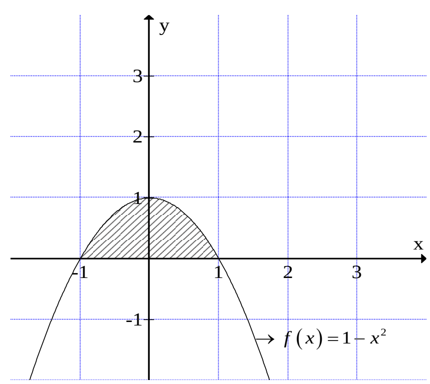
$=\pi\left(3x-\frac{x^2}{2}\right)\Big|_{-1}^{2}=\pi\cdot\frac{23}{2}$

៥.គណនាមាឌសូលីតបរិវត្តកំណត់បានពីរង្វិលចំនួន 360° ជុំ

វិញអ័ក្សអាប់ស៊ីសនៃផ្ទៃខណ្ឌដោយក្រាភិចតាងអនុគមន៍

$y=1-x^2$ និងអ័ក្ស $x'ox$, $x\in[-1,1]$ ។

ដំណោះស្រាយ



គណនាមាឌសូលីត

តាមរូបមន្តយើងបាន

$V=\pi\int_{-1}^{1}\left(1-x^2\right)^2 dx$

$=\pi\int_{-1}^{1}(1-2x^2+x^4)dx$

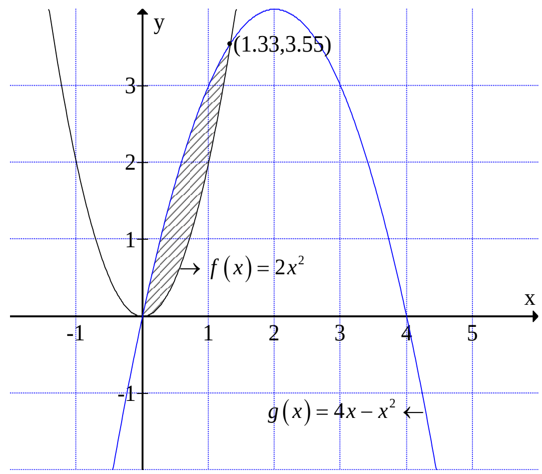
$=\pi\left(x-\frac{2x^3}{3}+\frac{x^5}{5}\right)\Big|_{-1}^{1}=\pi\frac{16}{15}$

៦.គណនាមាឌសូលីតបរិវត្តកំណត់បានពីរង្វិលជុំវិញអ័ក្ស $x'ox$

នៃផ្ទៃដែលខណ្ឌដោយក្រាភិចតាងអនុគមន៍ $f(x)=2x^2$ និង

អ័ក្ស $g(x)=4x-x^2$ ។

ដំណោះស្រាយ



គណនាមាឌសូលីត

រកចំនុចប្រសព្វរវាងខ្សែកោង

គេអោយ $2x^2=4x-x^2$

$\Rightarrow x_1=0\ ,x_2=4/3$ គេបាន

$$V=\pi\int_0^{4/3}\left(4x-x^2\right)^2dx-\pi\int_0^{4/3}4x^4dx$$

$$=\pi\int_0^{4/3}\left(16x^2-8x+x^4-4x^4\right)dx$$

$$=\pi\int_0^{4/3}\left(16x^2-8x^3-3x^4\right)dx$$

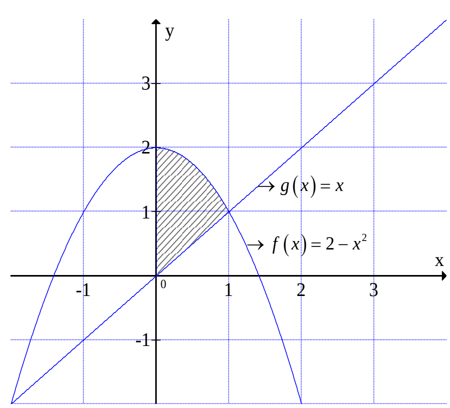
$$=\pi\left(\frac{16x^3}{3}-2x^4-\frac{3x^5}{5}\right)\Bigg|_0^{4/3}=\frac{512\pi}{135}$$

៧.គណនាមាឌសូលីតបរិវត្តកំណត់បានពីរង្វិលជុំវិញអ័ក្ស

$x'ox$ នៃផ្ទៃដែលខណ្ឌដោយក្រាភិចតាងអនុគមន៍ $g(x)=x$

និង $f(x)=2-x^2$ លើចន្លោះ $x\in[0,1]$ ។

ដំណោះស្រាយ



គណនាមាឌសូលីត

យើងពិនិត្យឃើញថា

$f(x)\geq g(x)\ ,x\in[0,1]$

$$\Rightarrow V=\pi\int_0^1\left[\left(2-x^2\right)^2-x^2\right]dx$$

$$=\pi\int_0^1\left(4-5x^2+x^4\right)dx$$

$$=\pi\left(4x-\frac{5x^3}{3}+\frac{x^5}{5}\right)\Bigg|_0^1=\frac{38\pi}{15}$$

៨.គណនាមាឌសូលីតបរិវត្តកំណត់បានពីរង្វិលចំនួន 180°

ជុំវិញអ័ក្ស $x'ox$ នៃផ្ទៃដែលខណ្ឌដោយក្រាភិចតាងអនុគមន៍

$f(x)=8-x^2$ និង $g(x)=x^2$ ។

ដំណោះស្រាយ

គណនាមាឌសូលីត

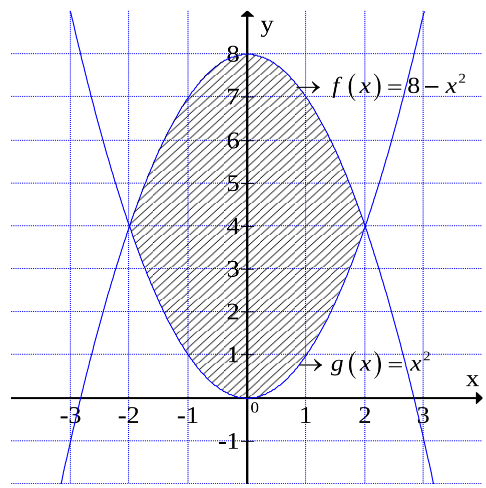
ចំនុចប្រសព្វនៃក្រាបតាងអនុគមន៍ទាំងពីរគឺ:

$8-x^2=x^2\Rightarrow x_1=-2\ ,x_2=2$

$\Rightarrow V=\pi\int_{-2}^2\left[\left(8-x^2\right)^2-x^4\right]dx$

$=\pi\int_{-2}^2\left(64-16x^2\right)dx$

$=\pi\left(64x-\frac{16x^3}{3}\right)\Bigg|_{-2}^2=\pi\frac{512}{3}$



៩.គណនាផ្ទៃក្រឡាដែលខណ្ឌដោយក្រាបតាងអនុគមន៍

$y=\sin\frac{\pi}{2}x$ និង $y=x^4$ ។

ដំណោះស្រាយ

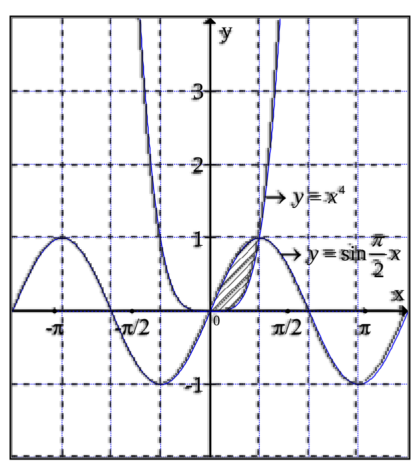
គណនាផ្ទៃក្រឡា

ចំនុចប្រសព្វរវាងខ្សែកោង

$\sin\frac{\pi}{2}x=x^4$ សមីការនេះផ្ទៀងផ្ទាត់

ពេល $x=0$ ឬ $x=1$ នោះយើងបាន

$S=\int_0^1\left(\sin\frac{\pi}{2}x-x^4\right)dx$

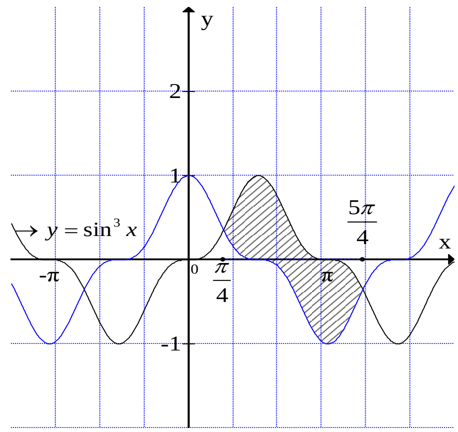


$$=\left(-\frac{\cos\frac{\pi}{2}x}{\frac{\pi}{2}}-\frac{x^5}{5}\right)\Bigg|_0^1=\frac{2}{\pi}-\frac{1}{5}$$

១០.គណនាផ្ទៃក្រឡាដែលខណ្ឌដោយក្រាបតាងអនុគមន៍

$y=\sin^3 x$ និង $y=\cos^3 x$ នៅចន្លោះ $x=\frac{\pi}{4}$ និង $x=\frac{5\pi}{4}$ ។

ដំណោះស្រាយ



គណនាផ្ទៃក្រឡា

ចំនុចប្រសព្វនៃខ្សែកោងគឺ:

$\sin^3 x=\cos^3 x$ សមីការនេះផ្ទៀងផ្ទាត់

ពេល $x=\frac{\pi}{4}$ ឬ $x=\frac{5\pi}{4}$

$S=\int_{\pi/4}^{5\pi/4}(\sin^3 x-\cos^3 x)dx$

$S=\int_{\pi/4}^{5\pi/4}\sin x\left(1-\cos^2 x\right)dx-\int_{\pi/4}^{5\pi/4}\cos x\left(1-\sin x\right)dx$

តាង $\begin{cases}u=\cos x\\ t=\sin x\end{cases}\Rightarrow\begin{cases}du=-\sin xdx\\ dt=\cos tdt\end{cases},\begin{cases}x=\frac{\pi}{4}\Rightarrow u=t=\frac{\sqrt{2}}{2}\\ x=\frac{5\pi}{4}\Rightarrow u=t=-\frac{\sqrt{2}}{2}\end{cases}$

$\Rightarrow S=\left[-\int_{\sqrt{2}/2}^{-\sqrt{2}/2}\left(1-u^2\right)du-\int_{\sqrt{2}/2}^{-\sqrt{2}/2}\left(1-t^2\right)dt\right]$

$=\int_{-\sqrt{2}/2}^{\sqrt{2}/2}\left(1-u^2\right)du+\int_{-\sqrt{2}/2}^{\sqrt{2}/2}\left(1-t^2\right)dt$

$=\int_{-\sqrt{2}/2}^{\sqrt{2}/2}(1-u^2+1-u^2)du$

$=2\int_{-\sqrt{2}/2}^{\sqrt{2}/2}\left(1-u^2\right)du$

$=2\left(u-\frac{u^3}{3}\right)\Bigg|_{-\sqrt{2}/2}^{\sqrt{2}/2}=2\left[\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{12}\right)+\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{12}\right)\right]=\frac{5\sqrt{2}}{3}$

១១.គណនាផ្ទៃក្រឡាដែលខណ្ឌដោយក្រាប c_1 និង c_2

អនុគមន៍ $y=\sin x$ $\left(0\le x\le\pi\right)$ និង $y=\sin 3x$ $\left(0\le x\le\pi\right)$ ។

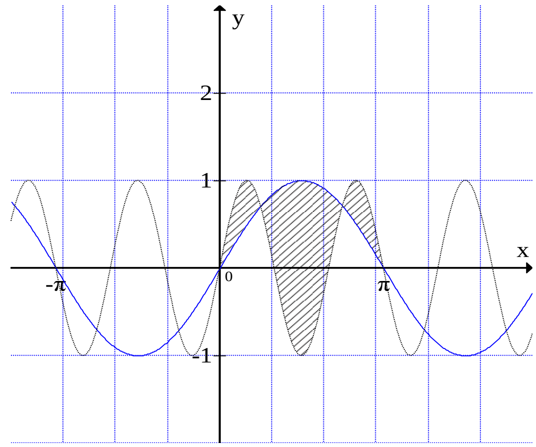
ដំណោះស្រាយ

គណនាផ្ទៃក្រឡា:

យើងមានសមីការអាប់ស៊ីស

$\sin x = \sin 3x$

$$\Rightarrow \left[\begin{array}{l} x = 0 \\ x = \pi/4 \\ x = 3\pi/4 \\ x = \pi \end{array}\right.$$



$$\Rightarrow S = 2\int_0^{\pi/4}(\sin 3x - \sin x)\,dx + \int_{\pi/4}^{3\pi/4}(\sin x - \sin 3x)\,dx$$

$$= 2\left(-\frac{\cos 3x}{3} + \cos x\right)\Big|_0^{\pi/4} + \left(-\cos x + \frac{\cos 3x}{3}\right)\Big|_{\pi/4}^{3\pi/4}$$

$$= 2\left[\left(\frac{\sqrt{2}}{6} + \frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{3} + 1\right)\right] + \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{6}\right) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{6}\right)$$

$$= 2\left(\frac{4\sqrt{2}}{6} - \frac{2}{3}\right) + \frac{8\sqrt{2}}{6}$$

$$= \frac{8\sqrt{2} - 4}{3}$$

១២.ពិនិត្យក្រាប $y = \sin 2x$ និង $y = \cos x$ ដែល $0 \le x \le \pi$

ក. រកកូអរដោនេចំនុចប្រសព្វនៃក្រាបទាំងនេះ។

ខ. រកផ្ទៃក្រឡា S ដែលខណ្ឌដោយក្រាបទាំងពីរ។

ដំណោះស្រាយ

ក. រកកូអរដោនេចំនុចប្រសព្វនៃក្រាបទាំងនេះ

ក្រាបនៃអនុគមន៍ទាំងពីរប្រសព្វគ្នាកាលណាវាមានតម្លៃស្មើគ្នា នោះយើងបាន $\sin 2x = \cos x$ លើចន្លោះ $[0, \pi]$

$$\Rightarrow x = \frac{\pi}{4}, x = \frac{\pi}{2}, x = \frac{3\pi}{4}$$

ដូចនេះក្រាបទាំងពីរប្រសព្វគ្នាត្រង់បីចំនុច:

$$\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right), \left(\frac{\pi}{2}, 0\right), \left(\frac{3\pi}{4}, \frac{-\sqrt{2}}{2}\right)$$

ខ.រកផ្ទៃក្រឡានៃ S

$$S = 2\int_0^{\pi/4}(\cos x - \sin 2x)\,dx + \int_{\pi/4}^{3\pi/4}(\sin 2x - \cos x)\,dx$$

$$= 2\left(\sin x + \frac{\cos 2x}{2}\right)\Big|_0^{\pi/4} - \left(\frac{\cos 2x}{2} + \sin x\right)\Big|_{\pi/4}^{3\pi/4}$$

$$= 2\left[\left(\frac{\sqrt{2}}{2} + 0\right) - \left(0 + \frac{1}{2}\right)\right] - 0$$

$$= 2\left(\frac{\sqrt{2}}{2} - \frac{1}{2}\right) = \sqrt{2} - 1$$

១៣. C ជាក្រាបតាង $f(x) = \ln(x+1)$ និង I ជាបន្ទាត់ប៉ះនឹង C ដែលមាន មេគុណប្រាប់ទិសស្មើនឹង $\frac{1}{2}$

ក. រកសមីការបន្ទាត់ប៉ះ I

ខ. គណនមាឌនៃសូលីតបរិវត្តកំណត់បានពីរង្វិលជុំវិញ អ័ក្ស $x'ox$ នៃផ្ទៃក្រឡាដែលខណ្ឌដោយក្រាប C ,I និងអ័ក្សអរដោនេ។

ដំណោះស្រាយ

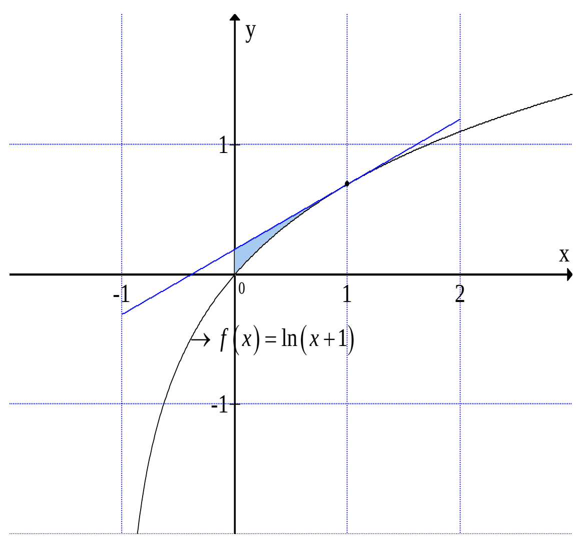
ក. រកសមីការបន្ទាត់ប៉ះ

តាង $I = \frac{1}{2}x + b$ ជាសមីការបន្ទាត់ប៉ះខ្សែកោងដែលមាន មេគុណប្រាប់ទិសស្មើ $\frac{1}{2}$ ។

ឧបមាថា I ប៉ះខ្សែកោង C ត្រង់ x_0

$\Rightarrow I = f'(x_0)(x - x_0) + f(x_0)$

ដោយ $f'(x) = \frac{1}{x+1}$



$$\Rightarrow I = \frac{1}{x_0+1}(x-x_0)+\ln(x_0+1)$$

$$= \frac{x}{x_0+1}-\frac{x_0}{x_0+1}+\ln(x_0+1)$$

$$\Rightarrow \begin{cases} \frac{1}{2}=\frac{1}{x_0+1} \\ b=-\frac{x_0}{x_0+1}+\ln(x_0+1) \end{cases} \Rightarrow \begin{cases} x_0=1 \\ b=-\frac{1}{2}+\ln(2) \end{cases}$$

$$\Rightarrow I = \frac{1}{2}x-\frac{1}{2}+\ln 2$$

ខ.គណនាមាឌសូលីតបរិវត្ត

តាមសំនួរ ក គេបាន

ក្រាបទាំងពីរប៉ះគ្នា

ត្រង់ $x_0=1$

$$\Rightarrow V = \pi\int_0^1\left(\frac{1}{2}x-\frac{1}{2}+\ln 2\right)^2 dx-\pi\int_0^1\left(\ln(x+1)\right)^2 dx$$

$$= \frac{\pi}{4}\int_0^1(x-1+2\ln 2)^2 dx-\pi\int_0^1\left(\ln(x+1)\right)^2 dx$$

$$= \frac{\pi}{4}I_1-\pi I_2$$

គណនា $I_1=\int_0^1(x-1+2\ln 2)^2 dx$

តាង $u=x-1+2\ln 2\Rightarrow du=dx$

$$\Rightarrow \int (x-1+2\ln 2)^2 dx=\int u^2du=\frac{u^3}{3}=\frac{(x-1+2\ln 2)^3}{3}$$

$$\Rightarrow I_1=\int_0^1(x-1+2\ln 2)^2 dx=\frac{1}{3}\left[(x-1+2\ln 2)^3\right]_0^1$$

$$= \frac{1}{3}\left(1-6\ln 2+12(\ln 2)^2\right)$$

គណនា $I_2=\int_0^1\left(\ln(x+1)\right)^2 dx$

$$t=\ln(x+1)\Rightarrow x+1=e^t\Rightarrow x=e^t-1,\begin{cases}x=0,\ t=0\\ x=1,\ t=\ln 2\end{cases}$$

តាង $dt=\dfrac{dx}{x+1}\Rightarrow dx=(x+1)dt=e^t dt$

$$\Rightarrow I_2=\int_0^{\ln 2}t^2e^t dt$$

តាង $u=t^2\Rightarrow du=2tdt\ \ ,dv=e^t dt\Rightarrow v=\int e^t dt=e^t$

$$\begin{aligned}\Rightarrow I_2&=\left[e^t t^2\right]_0^{\ln 2}-2\int_0^{\ln 2}te^t dt\\&=\left(e^{\ln 2}(\ln 2)^2\right)-2\left\{\left[te^t\right]_0^{\ln 2}-\int_0^{\ln 2}e^t dt\right\}\\&=2(\ln 2)^2-2\left(e^{\ln 2}\ln 2-e^t\Big|_0^{\ln 2}\right)\\&=2(\ln 2)^2-2(2\ln 2-1)\\&=2(\ln 2)^2-4\ln 2+2\end{aligned}$$

$$\begin{aligned}V&=\frac{\pi}{4}\cdot\frac{1}{3}\left(1-6\ln 2+12(\ln 2)^2\right)-\pi\left(2\left(\ln 2^2\right)-4\ln 2+2\right)\\&=-\pi(\ln 2)^2+\frac{7\pi}{2}\ln 2-\frac{23\pi}{12}\end{aligned}$$

១៩. តាង C ជាក្រាប $y=e^{2x-1}$ និង l ជាបន្ទាត់ប៉ះក្រាប C ត្រង់

ចនុច $P(1,e)$ ។

ក.រកសមីការបន្ទាត់ប៉ះ l ។

ខ. រកផ្ទៃក្រឡា S ដែលខណ្ឌដោយក្រាប C បន្ទាត់ប៉ះ l និងអ័ក្សអរដោនេ។

ដំណោះស្រាយ

ក.រកសមីការបន្ទាត់ប៉ះ

យើងមាន l ប៉ះ C ត្រង់ $P(1,e)$ យើងបាន $x_0=1$

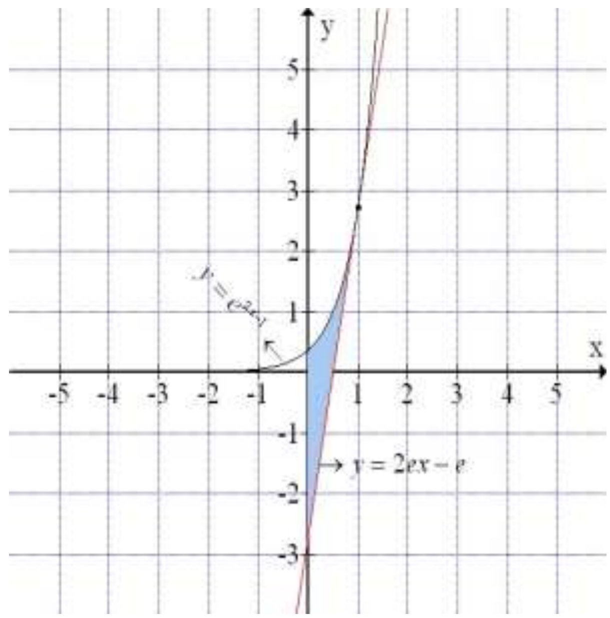
$\Rightarrow (l): y=f'(1)(x-1)+e$ តែ

$f(x)=e^{2x-1} \Rightarrow f'(x)=2e^{2x-1}$

$\Rightarrow f'(1)=2e$

$\Rightarrow y=2e(x-1)+e$

$=2ex-e$



ខ.រកផ្ទៃក្រឡា

$$S=\int_0^1\left(e^{2x-1}-e(2x-1)\right)dx$$

$$=\left[\frac{e^{2x-1}}{2}-e\left(x^2-x\right)\right]_0^1$$

$$=\frac{e}{2}-\frac{1}{2e}$$

១៥. តាង C ជាក្រាប $y=xe^x$,l_1 និង l_2 ជាបន្ទាត់ប៉ះ C ដែលគូស ចេញពីចនុច $\left(\frac{1}{2},0\right)$ ។

ក. រកសមីការបន្ទាត់ប៉ះ l_1, l_2 ។

ខ. រកផ្ទៃក្រឡា S ដែលខណ្ឌដោយក្រាប C និងបន្ទាត់ប៉ះដែលមាន នៅក្នុងសំនួរទី1។

ដំណោះស្រាយ

ក.រកសមីការបន្ទាត់ប៉ះ l_1 និង l_2

ឧបមាថា l ប៉ះខ្សែកោង C ត្រង់ x_0 នោះយើងបាន

$(l):y=f'(x_0)(x-x_0)+f(x_0)$

ដោយ $f(x)=xe^x \Rightarrow f'(x)=e^x+xe^x=e^x(1+x)$

$$f(x_0)=x_0e^{x_0},\ f'(x_0)=e^{x_0}(1+x_0)$$

$$\Rightarrow y=e^{x_0}(1+x_0)(x-x_0)+x_0e^{x_0}$$

$$=xe^{x_0}(1+x_0)-x_0e^{x_0}(1+x_0)+x_0e^{x_0}$$

$$=xe^{x_0}(1+x_0)-x_0^{\ 2}e^{x_0}$$

$$=e^{x_0}\left(x+x_0x-x_0^{\ 2}\right)$$

ម្យ៉ាងទៀតបន្ទាត់ l កាត់ចំនុច $P\left(\frac{1}{2},0\right)$

$$\Rightarrow e^{x_0}\left(\frac{1}{2}+\frac{1}{2}x_0-x_0^{\ 2}\right)=0$$

$$\Rightarrow \frac{1}{2}+\frac{1}{2}x_0-x_0^{\ 2}=0,\begin{cases}x_0=1\\ x_0=-\frac{1}{2}\end{cases}$$

+ករណី $x_0=1$

$(l_1): y=e(x+x-1)=2ex-e$

+ករណី $x_0=-\frac{1}{2}$

$$\Rightarrow (l_2): y=\frac{1}{\sqrt{e}}\left(x-\frac{x}{2}-\left(-\frac{1}{2}\right)^2\right)$$

$$=\frac{1}{\sqrt{e}}\left(\frac{x}{2}-\frac{1}{4}\right)$$

$$=\frac{x}{2\sqrt{e}}-\frac{1}{4\sqrt{e}}$$

ខ. រកផ្ទៃក្រឡា S

តាមសំនូរភាពយើងបានបន្ទាត់

l ប៉ះក្រាបត្រង់ពីរចំនុចគឺ:

$x_1=-\frac{1}{2}$, $x_2=1$ យើងបាន

$$S=\int_{-1/2}^{1/2}\left(xe^x-\frac{x}{2\sqrt{e}}+\frac{1}{4\sqrt{e}}\right)dx+\int_{1/2}^{1}\left(xe^x-2ex+e\right)dx$$

$$=\left(xe^x-e^x-\frac{x^2}{4\sqrt{e}}+\frac{x}{4\sqrt{e}}\right)\Bigg|_{-1/2}^{1/2}+\left(xe^x-e^x-ex^2+ex\right)\Big|_{1/2}^{1}$$

ដូចនេះ $\boxed{S=\frac{7}{4\sqrt{e}}-\frac{e}{4}}$ ឯកតាផ្ទៃ